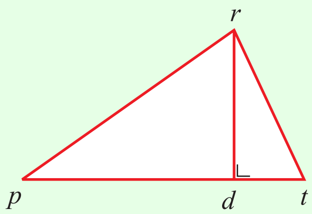
Geometry (Q 4, Paper 2)

2002

4 (a) The area of the triangle *rpt* is 30cm^2.
rd is perpendicular to *pt*.
Given that $|pt| = 12$cm, calculate $|rd|$.



(b) Prove that if three parallel lines make intercepts of equal length on a transversal, then they will also make intercepts of equal length on any other transversal.

(c) The triangle $a'b'c'$ is the image of the triangle *abc* under an enlargement.

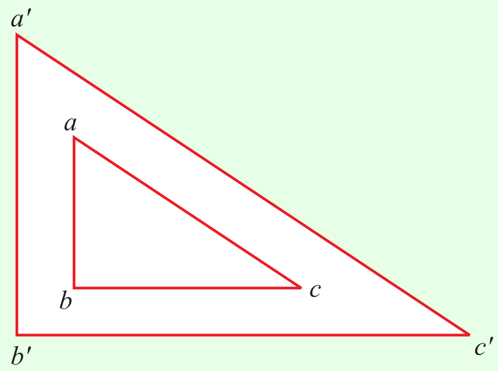
(i) Find, by measurement, the scale factor of the enlargement.

(ii) Copy the diagram and show how to find the centre of the enlargement.

(iii) Units are chosen so that $|bc| = 8$ units.

How many of these units is $|b'c'|$?

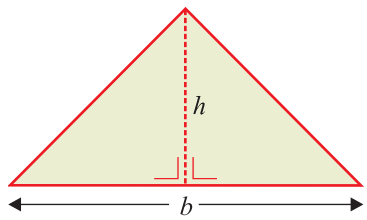
(iv) Find the area of triangle *abc*, given that the area of $a'b'c'$ is 84 square units.



Solution

4 (a)

4. Non Right-angled Triangles



b: Base
h: Height

$\boxed{A = \frac{1}{2}bh}$ **4**

$A = \frac{1}{2}|pt||rd| \Rightarrow 30 = \frac{1}{2}(12) \times |rd|$

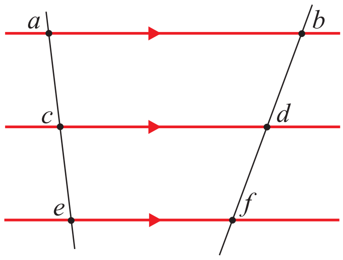
$\Rightarrow 30 = 6 \times |rd|$

$\therefore |rd| = 5$ cm

4 (b)

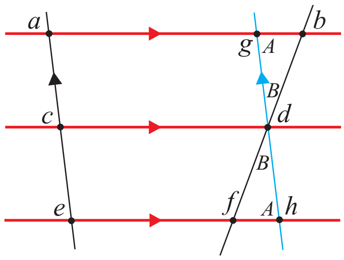
THEOREM 3: If three parallel lines make intercepts of equal length on a transversal, then they will make intercepts of equal lengths on any other transversal.

GIVEN: Three parallel lines *ab*, *cd* and *ef* such that *c* is on *ae* and *d* is on *bf* with $|ac| = |ce|$.



TO PROVE: $|bd| = |df|$.

CONSTRUCTION: Draw a line *gh* through *d* parallel to *ae* such that *g* is on *ab* and *h* is on *ef*.



PROOF: *acdg* is a parallelogram $\Rightarrow |ac| = |gd| = |ce|$

cehd is a parallelogram $\Rightarrow |ce| = |dh|$.

$\therefore |gd| = |dh|$.

Now Δgdb and Δfdh are congruent (**ASA**) because:

$|\angle bgd| = |\angle fhd| = A$ [Alternate angles]

$|\angle gdb| = |\angle fdh| = B$ [Vertically opposite angles]

$|gd| = |dh|$ [Already proved]

$\therefore |bd| = |df|$.

4 (c) (i)

Measure the length of side [*ab*] and the length of its image $[a'b']$.

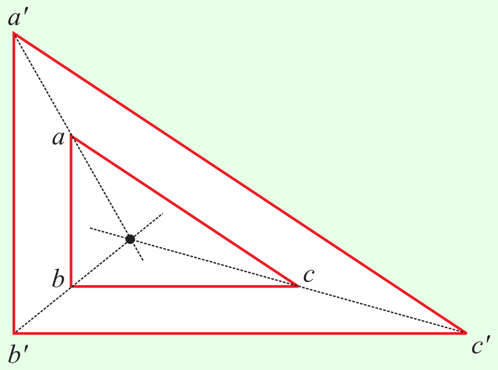
[*ab*] = 2 cm

$[a'b']$ = 4 cm

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \text{.......} \quad \textcircled{1}$$

$$\therefore k = \frac{|a'b'|}{|ab|} = \frac{4}{2} = 2$$

4 (c) (ii)



4 (c) (iii)

$$k = \frac{|b'c'|}{|bc|} \Rightarrow 2 = \frac{|b'c'|}{8}$$

$$\therefore |b'c'| = 2 \times 8 = 16 \text{ units}$$

4 (c) (iv)

$$\boxed{k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}} \quad \ldots\ldots\ \textbf{2}$$

$$k^2 = \frac{|\text{Area of triangle } a'b'c'|}{|\text{Area of triangle } abc|} \Rightarrow 2^2 = \frac{84}{|\text{Area of triangle } abc|}$$

$$\therefore |\text{Area of triangle } abc| = \frac{84}{2^2} = \frac{84}{4} = 21 \text{ square units}$$