## Geometry (Q 4, Paper 2)

2001
4 (a) Prove that the triangle with sides of lengths 10 units, 24 units and 26 units is right-angled.

(b) Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.
(c) (i) Draw a square with sides 7 cm and mark $o$, the point of intersection of the diagonals.
(ii) Draw the image of the square under the enlargement with centre $o$ and scale factor $\frac{1}{2}$.
(iii) Calculate the area of the image square.
(iv) Under another enlargement the area of the image of the square with sides 7 cm is $196 \mathrm{~cm}^{2}$.
What is the scale factor of this englargement?

## Solution

4 (a)
Theorem 7: (Converse of Pythagoras) If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides then the triangle has a right angle and this is opposite the longest side.

If you can show that $z^{2}=x^{2}+y^{2}$
$\Rightarrow \Delta a b c$ is a right-angled triangle and
$|\angle a b c|=90^{\circ}$ is opposite the longest side, $z$.

$26^{2}=676$
$10^{2}+24^{2}=100+576=676$
$\therefore 26^{2}=10^{2}+24^{2}$
Therefore, the triangle is a right-angled triangle.

## 4 (b)

Theorem 4: A line which is parallel to one side of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

Given: $\Delta a b c$ and line de parallel to $b c$ which cuts [ab] in the ratio m:n.


To Prove: $\frac{|a d|}{|d b|}=\frac{|a e|}{|e c|}=\frac{m}{n}$

Construction: Divide [ab] into $m$ and $n$ parts each of length $k$ so that $|a d|=m k$ and $|d b|=n k$.


Proof: According to theorem 3, [ac] is also divided into $m+n$ equal parts each of length $l$.

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\frac{|a e|}{|e c|}=\frac{m l}{n l}=\frac{m}{n}=\frac{|a d|}{|d b|} .
$$

## 4 (c) (i)

Construct a square $a b c d$ with side of length 7 cm .
Step 1. Using a ruler draw a side $a b$ of length 7 cm .
Step 2. Place the right-angle of a set square on point $a$ and draw a light line. Do the same at point $b$.

Step 3. Using the ruler draw a line, $a d$, of length 7 cm from point $a$ through the light line. Do the same at point $b$ drawing line $b c$.

Step 4. Complete the square by joining $c$ to $d$ to form line $c d$. Using your set square, make sure all the angles are right-angled. Using your ruler, make sure each side is of length 7 cm .


4 (c) (ii)


Note: All lengths shown are approximate. When you are doing the question the lengths must be the exact measure.

## 4 (c) (iii)

$A=3.5 \times 3.5=12.25 \mathrm{~cm}^{2}$

4 (c) (iv)

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\begin{equation*}
k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \tag{2}
\end{equation*}
$$

|Image area| $=196 \mathrm{~cm}^{2}$
|Object area| $=7 \times 7=49 \mathrm{~cm}^{2}$
Scale factor $k=$ ?
$k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \Rightarrow k^{2}=\frac{196}{49}=4$
$\therefore k=\sqrt{4}=2$

