## Geometry (Q 4, Paper 2)

1999
4 (a) $a b c$ is a triangle with $|a b|=8,|b c|=4$ and $|\angle a c b|=90^{\circ}$.
Calculate |ac|, correct to two places of decimals.

(b) Prove that the sum of the lengths of any two sides of a triangle is greater than that of the third side.
(c) The triangle ocd is the image of the triangle $o p q$ under the enlargement, centre $o$, with $|p q|=4,|o p|=5$ and $|c d|=9$.
(i) Find the scale factor of the enlargement.
(ii) Find $|p c|$.
(iii) The area of the triangle ocd is 60.75 square units. Find the area of the triangle opq.


## Solution

4 (a)
[C] Pythagoras Theorem
This theorem only applies to right-angled triangles.
Theorem 6: In a right-angled triangle the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

Triangle $a b c$ is right-angled with the $90^{\circ}$ angle at $b$.
The side opposite this angle is called the hypotenuse. This is the formula you use:

$$
z^{2}=x^{2}+y^{2}
$$


$|a b|^{2}=|a c|^{2}+|b c|^{2}$
$\Rightarrow 8^{2}=|a c|^{2}+4^{2}$
$\Rightarrow 64=|a c|^{2}+16$
$\Rightarrow|a c|^{2}=48$
$\therefore|a c|=\sqrt{48}=6.93$

## 4 (b)

Theorem 10: The sum of the lengths of any two sides of a triangle is greater than that of the third side.

Given: $\Delta a b c$


To Prove: $|a b|<|a c|+|b c|$

Construction: Draw a perpendicular [cd] onto [ab].


Proof: $\quad X<90^{\circ} \Rightarrow|a d|<|a c|$...... 1
$Y<90^{\circ} \Rightarrow|d b|<|b c| \ldots . . .2$
Adding 1 and 2: $|a d|+|d b|<|a c|+|b c|$
$\therefore|a b|<|a c|+|b c|$.

4 (c) (i)
Scale factor $k=\frac{\mid \text { Image length } \mid}{\mid \text { Object length } \mid}$
1
$k=\frac{|c d|}{|p q|}=\frac{9}{4}=2.25$

## 4 (c) (ii)

$k=\frac{|o c|}{|o p|} \Rightarrow 2.25=\frac{|o c|}{5}$

$\Rightarrow|o c|=5 \times 2.25=11.25$
$|o c|=|o p|+|p c| \Rightarrow 11.25=5+|p c|$
$\therefore|p c|=6.25$
4 (c) (iii)

$$
\begin{equation*}
k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \tag{2}
\end{equation*}
$$

$k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid}=\frac{\mid \text { Area of triangle } o c d \mid}{\mid \text { Area of triangle } o p q \mid}$
$\Rightarrow 2.25^{2}=\frac{60.75}{\mid \text { Area of triangle } o p q \mid}$
$\therefore \mid$ Area of triangle $o p q \left\lvert\,=\frac{60.75}{2.25^{2}}=12\right.$ square units

