## Geometry (Q 4, Paper 2)

## 1998

4 (a) In the triangle $a b c,|a d|=|b d|$, $|\angle a b d|=|\angle d b c|$ and $|\angle d a b|=48^{\circ}$. Find $|\angle d c b|$.

(b) Prove that if the lengths of two sides of a triangle are unequal, then the degreemeasures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.
(c) The triangle $x y z$ is the image of the triangle dgh under the enlargement, centre $o$, with $|d g|=8,|x z|=12$ and $|x y|=9$.

(i) Find the scale factor of the enlargement.
(ii) Find $|d h|$.
(iii) The area of the triangle $x y z$ is 27 square units. Find the area of the triangle dgh.

## Solution

4 (a)
Isosceles triangle: This is a triangle with two equal sides. The angles opposite the equal sides are equal.

$|\angle d a b|=|\angle d b a|=48^{\circ}$ [Isosceles triangle]
$|\angle d b a|=|\angle d b c|=48^{\circ}$ [Given]


## [E] Angles in a Triangle

Theorem 1: The sum of degree measure of the interior angles of a triangle is $180^{\circ}$.

Stated mathematically: $A+B+C=180^{\circ}$

$|\angle a d b|+48^{\circ}+48^{\circ}=180^{\circ}$ [Theorem 1]
$\therefore|\angle a d b|=180^{\circ}-96^{\circ}=84^{\circ}$

[A] Straight angles: $L$ is a straight line. The angles on $L$ add up to $180^{\circ}$.
$\therefore A+B+C=180^{\circ}$

$|\angle c d b|+84^{\circ}=180^{\circ}$ [Straight angle]
$\Rightarrow|\angle c d b|=180^{\circ}-84^{\circ}$
$\therefore|\angle c d b|=96^{\circ}$

$48^{\circ}+96^{\circ}+|\angle d c b|=180^{\circ}$ [Theorem 1]
$\Rightarrow|\angle d c b|=180^{\circ}-48^{\circ}-96^{\circ}$
$\therefore|\angle d c b|=36^{\circ}$


4 (b)
Theorem 9: If the lengths of a triangle are unequal, then the degree measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

Given: $\Delta a b c$ with $|a c|>|b c|$.


To Prove: $|\angle a b c|>|\angle c a b|$.

Construction: Mark a point $d$ on $[a c]$ such that $|c d|=|c b|$.


Proof: $|d c|=|c b| \Rightarrow|\angle c d b|=|\angle d b c|=X$ [Isosceles triangle]
$B=|\angle a b c|=X+Y$
$X=A+Y$ [Exterior Angle]
$\Rightarrow A=X-Y$
$\therefore B>A \Rightarrow|\angle a b c|=|\angle c a b|$.
Similarly, this is true for the other sides.

## 4 (c) (i)

$k=\frac{|x z|}{|d g|}=\frac{12}{8}=1.5$
4 (c) (ii)
$k=\frac{|x y|}{|d h|} \Rightarrow 1.5=\frac{9}{|d h|}$
$\therefore|d h|=\frac{9}{1.5}=6$
4 (c) (iii)

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k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \quad \ldots \ldots .
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$k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid}=\frac{\mid \text { Area of triangle } x y z \mid}{\mid \text { Area of triangle } d g h \mid}$
$\Rightarrow 1.5^{2}=\frac{27}{\mid \text { Area of triangle } d g h \mid}$
$\therefore \mid$ Area of triangle $d g h \left\lvert\,=\frac{27}{1.5^{2}}=12\right.$ square units

