## Geometry (Q 4, Paper 2)

## 1997

4 (a) Find the area of triangle $a b c$ if $|\angle a b c|=90^{\circ}$, $|a c|=26$ and $|b c|=10$.

(b) Prove that the products of the lengths of the sides of a triangle by the corresponding altitudes are equal.
(c) The triangle odc is the image of the triangle oab under an enlargement, centre $o$.
$|c d|=9$ and $|a b|=15$.
(i) Find the scale factor of the enlargement.
(ii) If the area of triangle $o a b$ is 87.5 square units, find the area of triangle odc.
(iii) Write down the area of the region $a b c d$.

## Solution



4 (a)
First, find the length of side $|a b|$ using Pythagoras.

## [C] Pythagoras Theorem

This theorem only applies to right-angled triangles.
Theorem 6: In a right-angled triangle the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

Triangle $a b c$ is right-angled with the $90^{\circ}$ angle at $b$. The side opposite this angle is called the hypotenuse. This is the formula you use:

$$
z^{2}=x^{2}+y^{2}
$$


$|a b|^{2}+10^{2}=26^{2}$
$\Rightarrow|a b|^{2}+100=676$
$\Rightarrow|a b|^{2}=676-100=576$
$\therefore|a b|=\sqrt{576}=24$


Area of a right-Angled triangle
You can find the area, $A$, by multiplying half the base, $b$, by the perpendicular height, $h$.

$$
\begin{array}{|l|l}
\hline & 1=\frac{1}{2} b h
\end{array} \ldots . . .4
$$


$A=\frac{1}{2}(10)(24)=120$ square units

4 (b)
Theorem 8: The products of the lengths of the sides of a triangle by the corresponding altitudes are equal.

Given: $\Delta a b c$ with altitudes $[a d]$ and $[c e]$.


To Prove: $|a d||b c|=|a b||c e|$.

Construction: Separate out $\Delta a b d$ and $\Delta e b c$.


Proof: $\Delta a b d$ and $\Delta e b c$ are equiangular because:
$|\angle a d b|=|\angle c e b|=90^{\circ}$
$|\angle a b d|=|\angle e b c|=B$ [Common]
$|\angle d a b|=|\angle e c b|=A$ [Remaining angle]
$\therefore \frac{|a d|}{|c e|}=\frac{|a b|}{|b c|} \Rightarrow|a d||b c|=|a b||c e|$

4 (c) (i)

$$
\text { Scale factor } k=\frac{\mid \text { Image length } \mid}{\mid \text { Object length } \mid} \ldots \ldots . . \text {. } 1
$$

$k=\frac{|c d|}{|b a|}=\frac{9}{15}=0.6$
4 (c) (ii)

$$
\begin{aligned}
& 4 \text { (c) (ii) } \quad k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \ldots . . . . .2 \\
& k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid}=\frac{\mid \text { Area of triangle } o d c \mid}{\mid \text { Area of triangle } o a b \mid}
\end{aligned}
$$

$\Rightarrow 0.6^{2}=\frac{\mid \text { Area of triangle odc } \mid}{87.5}$
$\therefore \mid$ Area of triangle odc $\mid=0.6^{2} \times 87.5=31.5$ square units

## 4 (c) (iii)

Area of $a b c d=$ Area of triangle $a a b-$ Area of triangle $o d c=87.5-31.5=56$ square units

