GEOMETRY (Q 4, PAPER 2)

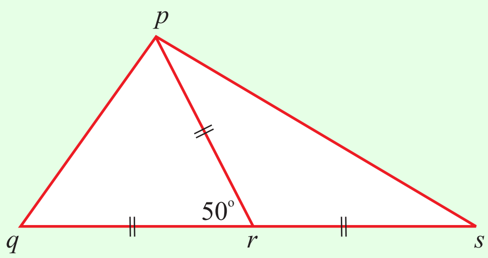
1996

4 (a) $|pr| = |qr| = |rs|$ and $|\angle prq| = 50^\circ$.

Find

(i) $|\angle pqr|$

(ii) $|\angle psr|$.

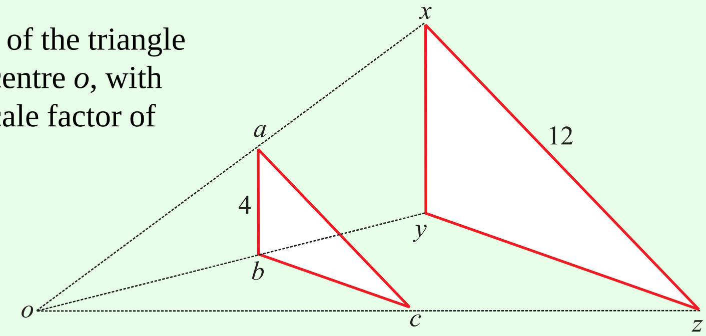


(b) Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

(c) The triangle *xyz* is the image of the triangle *abc* under the enlargement, centre *o*, with $|ab| = 4$ and $|xz| = 12$. The scale factor of the enlargement is 1.5.

(i) Find $|xy|$.

(ii) Find $|ac|$.

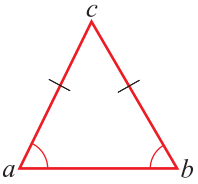


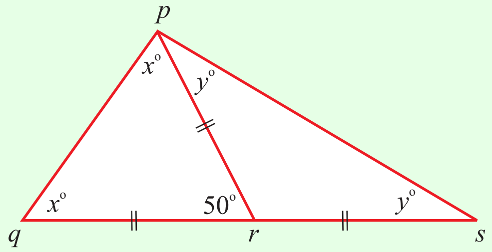
(iii) If the area of triangle *abc* is 12.2 square units, calculate the area of triangle *xyz*.

SOLUTION

4 (a) (i)

ISOSCELES TRIANGLE: This is a triangle with two equal sides. The angles opposite the equal sides are equal.





[E] **ANGLES IN A TRIANGLE**

THEOREM 1: The sum of degree measure of the interior angles of a triangle is 180°.

Stated mathematically: $A+B+C=180^\circ$

$|\angle pqr| = |\angle qpr| = x^\circ$ [Isosceles triangle]

$x^\circ + x^\circ + 50^\circ = 180^\circ$ [Theorem 1]

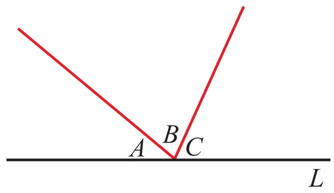
$\Rightarrow 2x^\circ = 130^\circ$

$\therefore x^\circ = |\angle pqr| = 65^\circ$

4 (a) (ii)

[A] **STRAIGHT ANGLES**: L is a straight line. The angles on L add up to 180°.

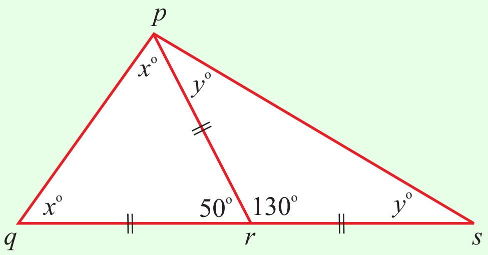
$\therefore A+B+C=180^\circ$



$50^\circ + |\angle prs| = 180^\circ$ [Straight angle]

$\Rightarrow |\angle prs| = 180^\circ - 50^\circ$

$\therefore |\angle prs| = 130^\circ$



$|\angle psr| = |\angle rps| = y^\circ$ [Isosceles triangle]

$y^\circ + y^\circ + 130^\circ = 180^\circ$ [Theorem 1]

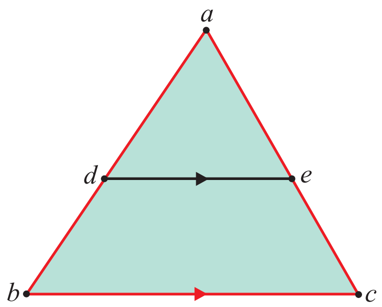
$\Rightarrow 2y^\circ = 50^\circ$

$\therefore y^\circ = |\angle psr| = 25^\circ$

4 (b)

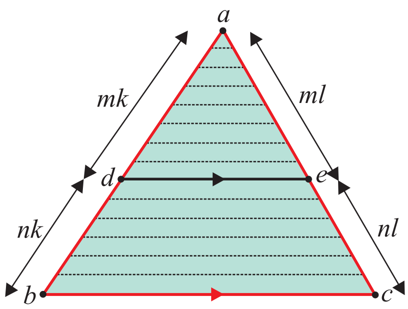
THEOREM 4: A line which is parallel to one side of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

GIVEN: Δabc and line de parallel to bc which cuts $[ab]$ in the ratio m:n.



TO PROVE: $\dfrac{|ad|}{|db|} = \dfrac{|ae|}{|ec|} = \dfrac{m}{n}$

CONSTRUCTION: Divide $[ab]$ into m and n parts each of length k so that $|ad| = mk$ and $|db| = nk$.



PROOF: According to theorem 3, $[ac]$ is also divided into $m + n$ equal parts each of length l.

$$\frac{|ae|}{|ec|} = \frac{ml}{nl} = \frac{m}{n} = \frac{|ad|}{|db|}.$$

4 (c) (i)

$$k=\frac{|xy|}{|ab|}\Rightarrow 1.5=\frac{|xy|}{4}$$

$$\boxed{\text{Scale factor } k=\frac{|\text{Image length}|}{|\text{Object length}|}} \;\ldots\ldots\; 1$$

$$\therefore |xy|=4\times 1.5=6$$

4 (c) (ii)

$$k=\frac{|xz|}{|ac|}\Rightarrow 1.5=\frac{12}{|ac|}$$

$$\therefore |ac|=\frac{12}{1.5}=8$$

4 (c) (iii)

$$\boxed{k^2=\frac{|\text{Image area}|}{|\text{Object area}|}} \;\ldots\ldots\; 2$$

$$k^2=\frac{|\text{Image area}|}{|\text{Object area}|}=\frac{|\text{Area of triangle } xyz|}{|\text{Area of triangle } abc|}$$

$$\Rightarrow 1.5^2=\frac{|\text{Area of triangle } xyz|}{12.2}$$

$$\therefore |\text{Area of triangle } xyz|=1.5^2\times 12.2=27.45 \text{ square units}$$