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DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

LESSON NO. 15: INTERSECTING GRAPHS

2005

8 Let $f(x)=\dfrac{1}{x-1},\ x\in\mathbf{R},\ x\neq 1.$

(i) Find $f(-3),\ f(-1.5),\ f(0.5),\ f(1.5),\ f(5)$.

(ii) Draw the graph of the function f from $x=-3$ to $x=5$.

(iii) On the same diagram, draw the graph of the function

$$g(x)=x+1$$

in the domain $-2\leq x\leq 2,\ x\in\mathbf{R}$.

(iv) Use your graphs to estimate the values of x for which $f(x)=g(x)$.

(v) Find, using algebra, the values of x for which $f(x)=g(x)$.

SOLUTION

8 (i)

$$f(x)=\frac{1}{x-1}$$

$$f(-3)=\frac{1}{(-3)-1}=-\frac{1}{4}=-0.25$$

$$f(-1.5)=\frac{1}{(-1.5)-1}=-\frac{1}{2.5}=-0.4$$

$$f(0.5)=\frac{1}{(0.5)-1}=-\frac{1}{0.5}=-2$$

$$f(1.5)=\frac{1}{(1.5)-1}=\frac{1}{0.5}=2$$

$$f(5)=\frac{1}{(5)-1}=\frac{1}{4}=0.25$$

8 (ii)

STEPS
1. Find the gap first by putting the bottom of the function equal to zero and solving for x.
2. Find other values of $f(x)$ by putting in values of x as given in the domain.
3. Plot these points by joining up smoothly and continuing towards the vertical line but never touching it.

1. Put $x-1=0\Rightarrow x=1$ is the asymptote.

2. You generated points in part (i) which you can use to draw the graph.

Points: $(-3,-0.25),\ (-1.5,-0.4),\ (-2,\ 0.5),\ (1.5,\ 2),\ (5,\ 0.25)$

3. Draw the graph.

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8 (iii)
$g(x) = x + 1$ is a straight line graph so you just need 2 points to draw the graph. Use the end values of the domain.

$x=-2: g(x)=x+1 \Rightarrow g(-2)=(-2)+1=-1 \Rightarrow (-2, -1)$ is a point.

$x=2: g(x)=x+1 \Rightarrow g(2)=(2)+1=3 \Rightarrow (2, 3)$ is a point.

Plot these two points using the same axes and draw a straight line through them.

8 (iv)
Find out where the two graphs intersect and read off the x values.

You can see that $x = 1.4$ and $x=-1.4$.

8 (v)

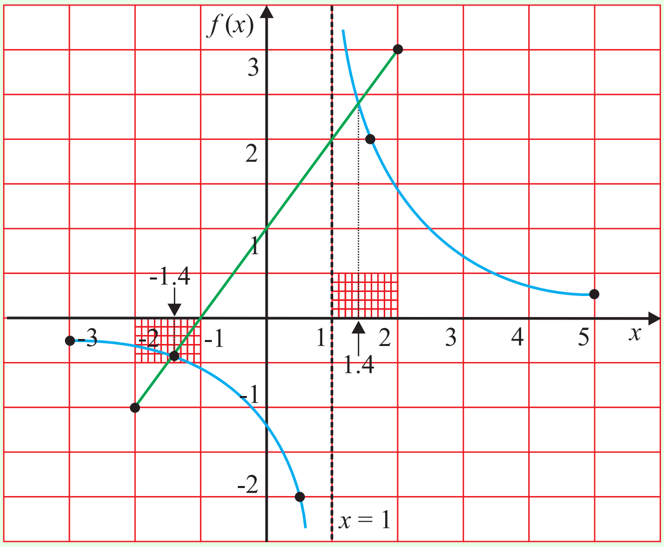
$f(x)=g(x) \Rightarrow \dfrac{1}{x-1}=x+1$ [Multiply across by $(x-1)$.]

$\Rightarrow 1=(x+1)(x-1)$ [Multiply out the brackets.]

$\Rightarrow 1=x^2-1$

$\Rightarrow 2=x^2$

$\Rightarrow x=\pm\sqrt{2}$



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8 (b) (i) Draw the graph of

$$g(x)=\tfrac{1}{x} \text{ for } -3 \le x \le 3,\ x \in \mathbf{R} \text{ and } x \neq 0.$$

(ii) Using the same axes and the same scales, draw the graph of

$$h(x) = x + 1 \text{ for } -3 \le x \le 3,\ x \in \mathbf{R}.$$

(iii) Use your graphs to estimate the values of x for which

$$\tfrac{1}{x}=x+1.$$

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SOLUTION

8 (b) (i)

DRAWING RECIPROCAL GRAPHS

STEPS
1. Find the gap first by putting the bottom of the function equal to zero and solving for x.
2. Find other values of $f(x)$ by putting in values of x as given in the domain.
3. Plot these points by joining up smoothly and continuing towards the vertical line but never touching it.

1. Put $x = 0$. The line $x = 0$ (i.e. the y-axis) represents the gap or asymptote.

2. $x=-3: f(-3)=\frac{1}{-3}=-\frac{1}{3}=-0.33 \Rightarrow (-3, -0.33)$

$x=-2: f(-2)=\frac{1}{-2}=-\frac{1}{2}=-0.5 \Rightarrow (-2, -0.5)$

$x=-1: f(-1)=\frac{1}{-1}=-1 \Rightarrow (-1, -1)$

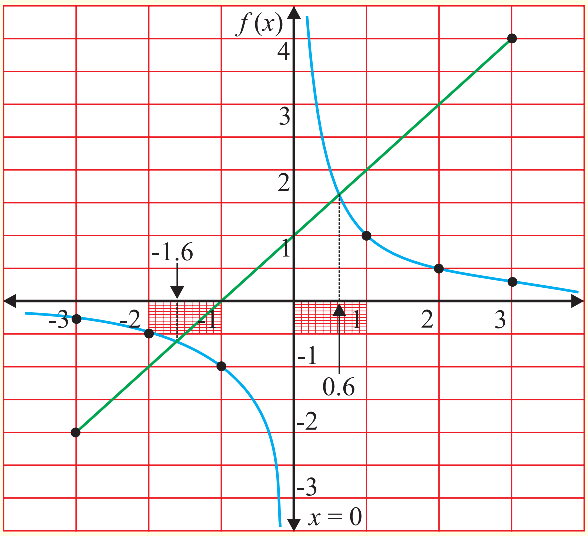
$x=0$: This is the gap where the function is not defined.

$x=1: f(1)=\frac{1}{1}=1 \Rightarrow (1, 1)$

$x=2: f(2)=\frac{1}{2}=0.5 \Rightarrow (2, 0.5)$

$x=3: f(3)=\frac{1}{3}=0.33 \Rightarrow (3, 0.33)$

3. Plot the reciprocal graph.



8 (b) (ii)

As these graphs are straight lines, you only need to plot the first and last points in the domain.

$h(x) = x+1$

$\Rightarrow h(-3) = -3+1 = -2 \Rightarrow (-3, -2)$ is a point on the straight line.

$\Rightarrow h(3) = 3+1 = 4 \Rightarrow (3, 4)$ is a point on the straight line.

8 (b) (iii)

Read off the x values of where the graphs intersect.

$\therefore x = -1.6, 0.6$

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8 Let $f(x)=2x^3-5x^2-4x+3$ for $x\in\mathbf{R}$.

(i) Complete the table

x	-1.5	-1	0	1	2	3	3.5
$f(x)$	-9						13.5

(ii) Find the derivative of $f(x)$.
Calculate the co-ordinates of the local minimum and show that the co-ordinates of the local maximum are $(-\frac{1}{3}, \frac{100}{27})$.

(iii) Draw the graph of

$$f(x)=2x^3-5x^2-4x+3$$

for $-1.5\le x\le 3.5$.

(iv) Write the equation $2x^3-5x^2-6x+6=0$ in the form

$$2x^3-5x^2-4x+3=ax+b,\quad a, b\in\mathbf{Z}.$$

Hence, use your graph to estimate the solutions of the equation

$$2x^3-5x^2-6x+6=0.$$

SOLUTION

8 (i)

$f(x)=2x^3-5x^2-4x+3$

$\Rightarrow f(-1.5)=2(-1.5)^3-5(-1.5)^2-4(-1.5)+3=-9$ [Use your calculator.]

Find the other values of x in the same way.

x	-1.5	-1	0	1	2	3	3.5
$f(x)$	-9	0	3	-4	-9	0	13.5

8 (ii) **STEPS FOR FINDING THE LOCAL MAXIMUM AND LOCAL MINIMUM OF A FUNCTION:**

STEPS

1. Differentiate the function to find $\frac{dy}{dx}$. Differentiate again to find $\frac{d^2y}{dx^2}$.

2. Set $\frac{dy}{dx}=0$ and solve for x to find the turning points.

3. Substitute the turning points into $\frac{d^2y}{dx^2}$ to decide if they are a local maximum or a local minimum.

4. Find the y coordinates of the turning points by substituting the x values back into the equation of the original function.

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1. $y = f(x) = 2x^3 - 5x^2 - 4x + 3$

$$\frac{dy}{dx} = f'(x) = 6x^2 - 10x - 4$$

$$\frac{d^2y}{dx^2} = f''(x) = 12x - 10$$

2. $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 10x - 4 = 0$

$\Rightarrow 3x^2 - 5x - 2 = 0$

$\Rightarrow (3x+1)(x-2) = 0$

$\therefore x = -\frac{1}{3}, 2$

3. $\left(\frac{d^2y}{dx^2}\right)_{x=-\frac{1}{3}} = 12(-\tfrac{1}{3}) - 10 = -4 - 10 = -14 < 0$

$\left(\frac{d^2y}{dx^2}\right)_{x=2} = 12(2) - 10 = 24 - 10 = 14 > 0$

Local Maximum: $\left(\frac{d^2y}{dx^2}\right)_{TP} < 0$

Local Minimum: $\left(\frac{d^2y}{dx^2}\right)_{TP} > 0$

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4. $x = -\frac{1}{3} : y = 2(-\tfrac{1}{3})^3 - 5(-\tfrac{1}{3})^2 - 4(-\tfrac{1}{3}) + 3 = \tfrac{100}{27} \Rightarrow (-\tfrac{1}{3}, \tfrac{100}{27})$ is a local maximum.

$x = 2 : y = 2(2)^3 - 5(2)^2 - 4(2) + 3 = -9 \Rightarrow (2, -9)$ is a local minimum.

8 (iii)

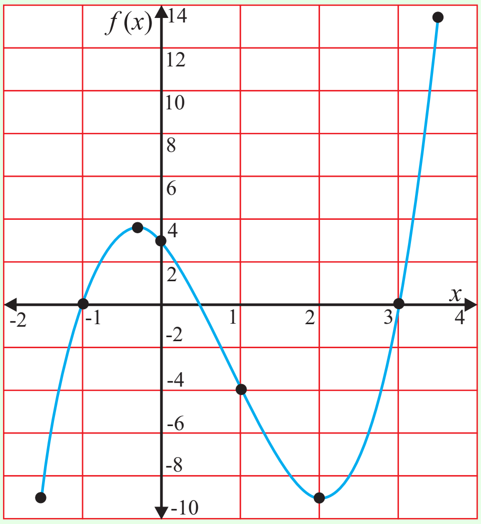
Draw the cubic graph using the information from the previous parts.

Points: $(-1.5, -9), (-1, 0), (0, 3), (1, -4)$

$(2, -9), (3, 0), (3.5, 13.5)$

Local maximum: $(-\tfrac{1}{3}, \tfrac{100}{27}) = (-0.33, 3.7)$

Local minimum: $(2, -9)$



8 (iv)

$2x^3 - 5x^2 - 6x + 6 = 0$

$\Rightarrow 2x^3 - 5x^2 - 4x + 3 = 2x - 3$

Let $h(x) = 2x - 3$

$\therefore f(x) = h(x)$.

$h(x)$ is a straight line. You want to find where the straight line and the cubic graph intersect. Graph $h(x)$ by using the first and last points of the domain.

$h(-1.5) = 2(-1.5) - 3 = -3 - 3 = -6$

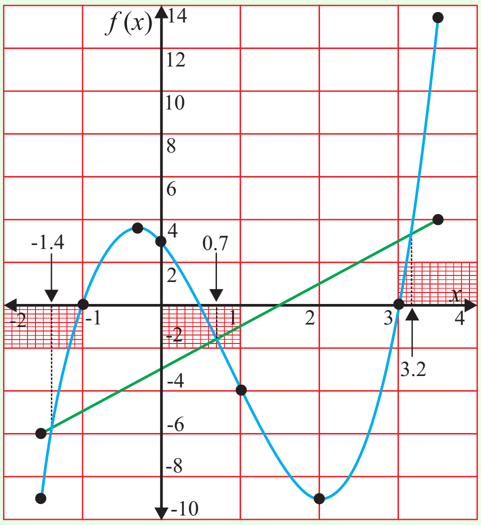
$\Rightarrow (-1.5, -6)$ is a point on the graph.

$h(3.5) = 2(3.5) - 3 = 7 - 3 = 4$

$\Rightarrow (3.5, 4)$ is a point on the graph.

You can see the x values of the places where the graphs intersect:

$\therefore x = -1.4, 0.7, 3.2$



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8 (c) Draw a graph of

$$g(x) = \frac{1}{x+2}$$

for $0 \le x \le 4,\ x \in \mathbf{R}$.

Using the same axes and the same scales draw the graph of

$$h(x) = x - 2.$$

Show how your graphs may be used to estimate the value of $\sqrt{5}$.

SOLUTION

DRAWING RECIPROCAL GRAPHS

STEPS

1. Find the gap first by putting the bottom of the function equal to zero and solving for x.
2. Find other values of $f(x)$ by putting in values of x as given in the domain.
3. Plot these points by joining up smoothly and continuing towards the vertical line but never touching it.

1. Put $x+2=0 \Rightarrow x=-2$. This line represents the gap or asymptote.

2. $g(x) = \dfrac{1}{x+2}$

$g(0) = \dfrac{1}{0+2} = \dfrac{1}{2} \Rightarrow (0, \tfrac{1}{2})$ is a point on the graph.

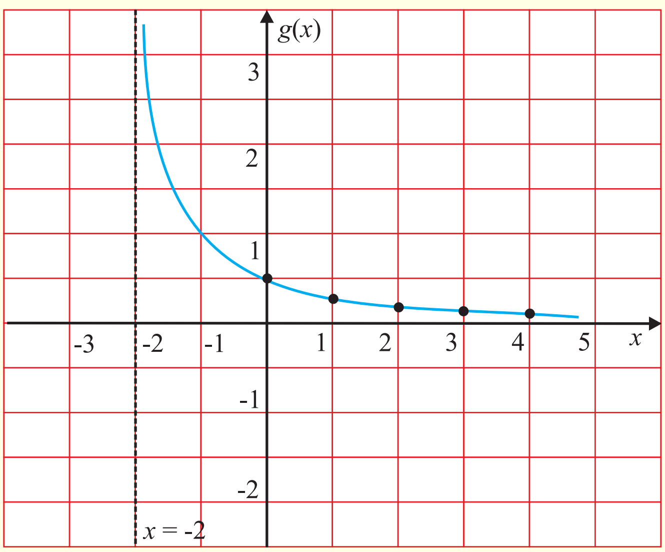
$g(1) = \dfrac{1}{1+2} = \dfrac{1}{3} \Rightarrow (0, \tfrac{1}{3})$ is a point on the graph.

$g(2) = \dfrac{1}{2+2} = \dfrac{1}{4} \Rightarrow (0, \tfrac{1}{4})$ is a point on the graph.

$g(3) = \dfrac{1}{3+2} = \dfrac{1}{5} \Rightarrow (0, \tfrac{1}{5})$ is a point on the graph.

$g(4) = \dfrac{1}{4+2} = \dfrac{1}{6} \Rightarrow (0, \tfrac{1}{6})$ is a point on the graph.

3. Plot the graph.



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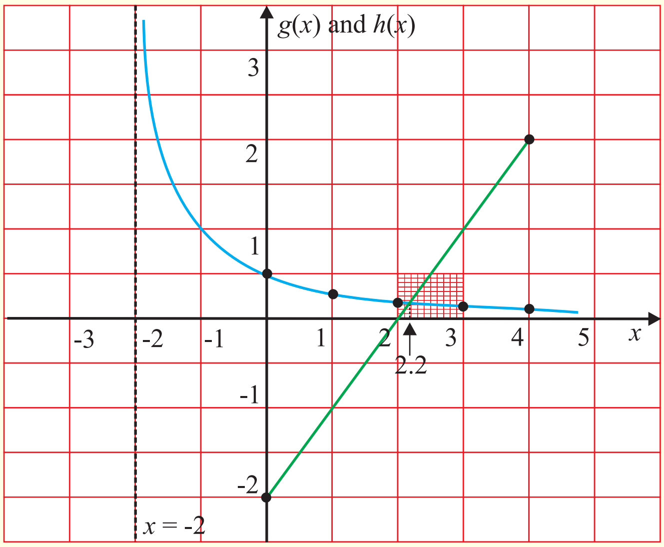
The graph of $h(x) = x - 2$ is a straight line graph. You just need to get 2 points on it to graph it. Choose the end points of the domain.

$x = 0 : h(0) = (0) - 2 = -2 \Rightarrow (0, -2)$ is a point on the graph.

$x = 4 : h(4) = (4) - 2 = 2 \Rightarrow (4, 2)$ is a point on the graph.



Steps
1. Draw each graph on the same diagram using the same scales.
2. Mark the points of intersection and read off their x value.
3. Solve $f(x) = g(x)$ exactly by algebra.
4. Hence, estimate the solution.

1. Graphs drawn as above.

2. $x = 2.2$ is their point of intersection.

3. $g(x) = h(x) \Rightarrow \dfrac{1}{x+2} = x - 2$

$\Rightarrow 1 = (x-2)(x+2) \Rightarrow 1 = x^2 - 4 \Rightarrow x^2 = 5$

$\therefore x = \sqrt{5}$

4. $\therefore \sqrt{5} \approx 2.2$