Differentiation & Functions (Q 6, 7 & 8, Paper 1)

Lesson No. 12: Quadratic Graphs

2006

6 (c) Let $f(x) = 3 + 8x - 2x^2,\ x \in \mathbf{R}$.

(i) Find the co-ordinates of the point at which the curve $y = f(x)$ cuts the y-axis.

(ii) Find the value of x for which $f(x)$ is a maximum.

(iii) For what range of values of x is $f'(x) > 4$?

Solution

6 (c) (i)

$f(x) = 3 + 8x - 2x^2$

Graph cuts y axis: Put $x = 0$.

$\Rightarrow f(x) = 3 + 8(0) - 2(0)^2 = 3$

The graph cuts the y-axis at (0, 3).

6 (c) (ii)

$f(x) = 3 + 8x - 2x^2$

$\Rightarrow f'(x) = 0 + 8 - 2 \times 2x$

$\Rightarrow f'(x) = 8 - 4x$

Turning Point $\Rightarrow \dfrac{dy}{dx} = 0$ 6

To find the value of x of the turning point which you are told is a maximum, put $f'(x) = 0$.

To find the turning points set $\dfrac{dy}{dx} = 0$ and solve for x.

$f'(x) = 0 \Rightarrow 8 - 4x = 0$

$\Rightarrow 2 - x = 0$

$\Rightarrow x = 2$

6 (c) (iii)

$f'(x) > 0 \Rightarrow 8 - 4x > 4$

$\Rightarrow 2 - x > 1$

$\Rightarrow -x > -1$ [Multiply across by a negative number. Remember to reverse the inequality.]

$\Rightarrow x < 1$

2003

6 (c) Let $f(x)=3-5x-2x^2,\ x\in\mathbf{R}$.

(i) Find $f'(x)$, the derivative of $f(x)$, and hence find the co-ordinates of the local maximum point of the curve $y = f(x)$.

(ii) Solve the equation $f(x) = 0$.

(iii) Use your answers from parts (i) and (ii) to sketch the graph of

$f: x\to 3-5x-2x^2$, showing scaled and labelled axes.

SOLUTION

6 (c) (i)

STEPS FOR FINDING THE LOCAL MAXIMUM AND LOCAL MINIMUM OF A FUNCTION:

STEPS

1. Differentiate the function to find $\frac{dy}{dx}$. Differentiate again to find $\frac{d^2y}{dx^2}$.

2. Set $\frac{dy}{dx}=0$ and solve for x to find the turning points.

3. Substitute the turning points into $\frac{d^2y}{dx^2}$ to decide if they are a local maximum or a local minimum.

4. Find the y coordinates of the turning points by substituting the x values back into the equation of the original function.

1. $y=f(x)=3-5x-2x^2$

$$\frac{dy}{dx}=f'(x)=-5-4x$$

$$\frac{d^2y}{dx^2}=f''(x)=-4$$

2. $\frac{dy}{dx}=0\Rightarrow -5-4x=0$

$\Rightarrow -5=4x$

$\therefore x=-\frac{5}{4}$

3. You are told that this point is a maximum.

4. $y=f(-\frac{5}{4})=3-5(-\frac{5}{4})-2(-\frac{5}{4})^2$

$=3+\frac{25}{4}-2(\frac{25}{16})=3+\frac{25}{4}-\frac{25}{8}=\frac{49}{8}\Rightarrow(-\frac{5}{4},\ \frac{49}{8})$ is the local maximum.

6 (c) (ii)

$f(x)=0\Rightarrow 3-5x-2x^2=0$

$\Rightarrow 2x^2+5x-3=0$

$\Rightarrow (2x-1)(x+3)=0$

$\Rightarrow x=-3,\ \frac{1}{2}$

Graph cuts x-axis: Put $f(x) = 0$.

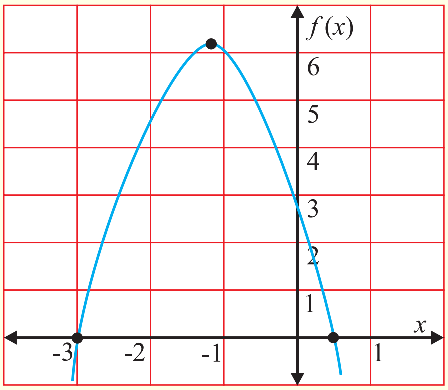
CONT....

6 (c) (iii)

From part **(i)** you have found out the coordinates of the local maximum: $(-\frac{5}{4}, \frac{49}{8})$

From part **(ii)** you have found out where the graph cuts the x-axis: $(-3, 0)$, $(\frac{1}{2}, 0)$

Draw the quadratic graph using these points.



1996

6 (b) Let $g(x) = x^2 + bx + c,\ x \in \mathbf{R}$.

The solutions of $g(x) = 0$ are symmetrical about the line $x = 1$.

If $x = -3$ is one solution of $g(x) = 0$, find the other solution.

Find the value of b and the value of c.

Solution

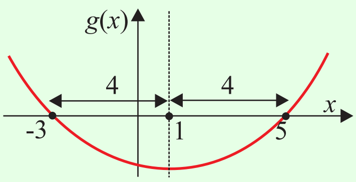
$g(x) = x^2 + bx + c$ is a quadratic graph as shown to the right.

The line $x = 1$ is its axis of symmetry.

-3 is a solution (root) which is a place where the graph cuts the x-axis.

You can see from the diagram that the other solution is $x = 5$.

If you substitute solutions for x in the function $g(x)$, you get zero.



$x = -3 : g(-3) = (-3)^2 + b(-3) + c = 0 \Rightarrow 9 - 3b + c = 0$

$\therefore -3b + c = -9 \ldots\mathbf{(1)}$

$x = 5 : g(5) = (5)^2 + b(5) + c = 0 \Rightarrow 25 + 5b + c = 0$

$\therefore 5b + c = -25 \ldots\mathbf{(2)}$

Solve equation (**1**) and (**2**) simultaneously.

$$\boxed{\begin{array}{r} -3b + c = -9\ldots\mathbf{(1)}(\times -1) \\ 5b + c = -25..\mathbf{(2)} \end{array}} \rightarrow \boxed{\begin{array}{l} 3b - c = 9 \\ 5b + c = -25 \\ \hline 8b \quad = -16 \Rightarrow b = -2 \end{array}}$$

Substitute this value of b into Eqn. (**2**).

$5(-2) + c = -25 \Rightarrow -10 + c = -25 \Rightarrow c = -15$