DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

LESSON NO. 15: INTERSECTING GRAPHS

2005

8   Let $f(x) = \dfrac{1}{x-1}$, $x \in \mathbf{R}$, $x \neq 1$.

   (i) Find $f(-3)$, $f(-1.5)$, $f(0.5)$, $f(1.5)$, $f(5)$.

   (ii) Draw the graph of the function f from $x = -3$ to $x = 5$.

   (iii) On the same diagram, draw the graph of the function
$$g(x) = x + 1$$
   in the domain $-2 \leq x \leq 2$, $x \in \mathbf{R}$.

   (iv) Use your graphs to estimate the values of x for which $f(x) = g(x)$.

   (v) Find, using algebra, the values of x for which $f(x) = g(x)$.

2000

8   (b) (i) Draw the graph of
$$g(x) = \tfrac{1}{x} \text{ for } -3 \leq x \leq 3,\ x \in \mathbf{R} \text{ and } x \neq 0.$$
     (ii) Using the same axes and the same scales, draw the graph of
$$h(x) = x + 1 \text{ for } -3 \leq x \leq 3,\ x \in \mathbf{R}.$$
     (iii) Use your graphs to estimate the values of x for which
$$\tfrac{1}{x} = x + 1.$$

1999

8   Let $f(x) = 2x^3 - 5x^2 - 4x + 3$ for $x \in \mathbf{R}$.

(i) Complete the table

x	−1.5	−1	0	1	2	3	3.5
$f(x)$	−9						13.5

(ii) Find the derivative of $f(x)$.
Calculate the co-ordinates of the local minimum and show that the co-ordinates
of the local maximum are $(-\frac{1}{3}, \frac{100}{27})$.

(iii) Draw the graph of
$$f(x) = 2x^3 - 5x^2 - 4x + 3$$
for $-1.5 \le x \le 3.5$.

(iv) Write the equation $2x^3 - 5x^2 - 6x + 6 = 0$ in the form
$$2x^3 - 5x^2 - 4x + 3 = ax + b, \quad a, b \in \mathbf{Z}.$$
Hence, use your graph to estimate the solutions of the equation
$$2x^3 - 5x^2 - 6x + 6 = 0.$$

1997

8   (c) Draw a graph of
$$g(x) = \frac{1}{x+2}$$
for $0 \le x \le 4,\ x \in \mathbf{R}$.
Using the same axes and the same scales draw the graph of
$$h(x) = x - 2.$$
Show how your graphs may be used to estimate the value of $\sqrt{5}$.

ANSWERS

2005   8   (i) $-0.25, -0.4, -2, 2, 0.25$ (iv) ± 1.4 (v) $\pm\sqrt{2}$

2000   8   (b) (iii) $x = -1.6, 0.6$

1999   8   (i)

x	−1.5	−1	0	1	2	3	3.5
$f(x)$	−9	0	3	−4	−9	0	13.5

(ii) $6x^2 - 10x - 4$; $(2, -9)$

(iv) $a = 2, b = -3$; $x = -1.4, 0.7, 3.2$