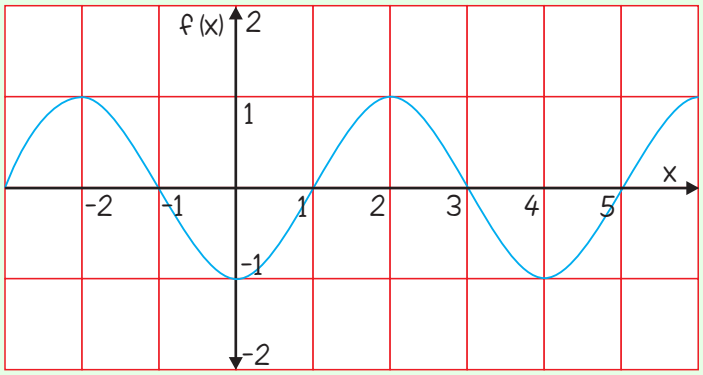
Differentiation & Functions (Q 6, 7 & 8, Paper 1)

2011

6. **(a)** $f : x \to f(x)$ is a periodic function defined for $x \in \mathbb{R}$.
The period is as indicated in the diagram.



(i) Write down the period and the range of the function.

(ii) Find $f(71)$.

(b) **(i)** Differentiate $(4x-1)(3-2x^2)$ with respect to x and simplify your answer.

(ii) Given that $y = \dfrac{1}{x^2-3x}$, $x \neq 3$, find the range of values of x for which $\dfrac{dy}{dx} < 0$.

(c) Let $f(x) = 2x + \dfrac{1}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$.

(i) Find the equation of the tangent to the curve $y = f(x)$ at the point $P(1, 3)$.

(ii) Q is another point on the curve $y = f(x)$ such that the tangent at Q is parallel to the tangent at P. Find the co-ordinates of Q.

Answers

6 (a) (i) Period = 4, Range = [–1, 1] (ii) 0

(b) (i) $-4(6x^2 - x - 3)$ (ii) $x > \frac{3}{2}$, $x \neq 3$, $x \in \mathbb{R}$

(c) (i) $x - y + 2 = 0$ (ii) $Q(-1, -3)$

7. **(a)** Differentiate $x^3 - 7x^2 + 6x$ with respect to x.

(b) **(i)** Differentiate $\dfrac{3x+1}{x-2}$ with respect to x.

Write your answer in the form $\dfrac{k}{(x-2)^n}$, where $k,\ n \in \mathbb{Z}$.

(ii) Given that $y = (x^2 - 2x - 9)^4$, find the value of $\dfrac{dy}{dx}$ when $x = -2$.

(c) A ball is rolled in a straight line along a surface.
The distance, s metres, the ball travels is given by

$$s = 18t - 2t^2$$

where t is the time in seconds from the instant the ball begins to move.

(i) Find the speed of the ball after 3 seconds.

(ii) How far is the ball from the starting point when it stops moving?

(iii) Show that the speed of the ball decreases at a constant rate while it is moving.

ANSWERS

7 (a) $3x^2 - 14x + 6$

(b) (i) $-\dfrac{7}{(x-2)^2}$ (ii) 24

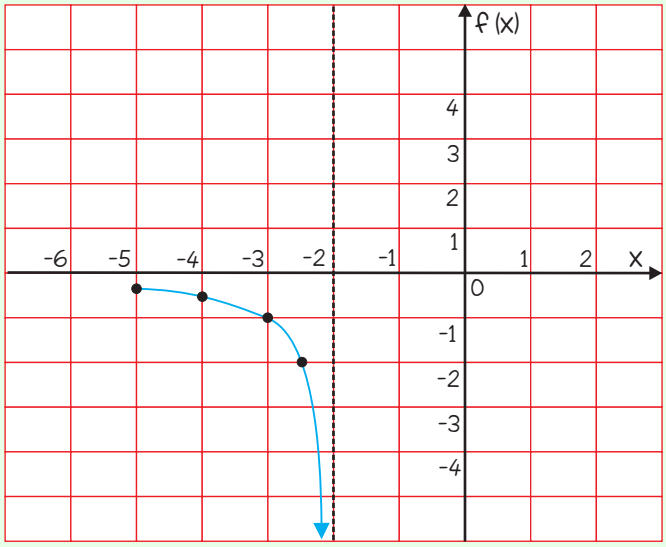
(c) (i) 6 m/s (ii) 40.5 m

8. Let $f(x) = \dfrac{1}{x+2}$, where $x \in \mathbb{R}$ and $x \neq -2$.

(i) Copy and complete the following table:

x	–5	–4	–3	–2.5	–1.5	–1	0	1
$f(x)$		–0.5	–1	–2				

(ii) The diagram shows part of the graph of the function f.
Copy and complete the graph from $x = -5$ to $x = 1$.



(iii) On the same diagram, draw the graph of the function $g(x) = x + 2$ in the domain $-5 \leq x \leq 1$, where $x \in \mathbb{R}$.

(iv) Use your graphs to estimate the range of values of x for which $f(x) \leq g(x)$.

(v) Prove that the curve $y = f(x)$ has no turning points.

ANSWERS

8 (i)

x	–5	–4	–3	–2.5	–1.5	–1	0	1
$f(x)$	$-\frac{1}{3}$	–0.5	–1	–2	2	1	$\frac{1}{2}$	$\frac{1}{3}$

(iii)

x	–5	–4	–3	–2	–1	0	1	2
$g(x)$	–3	–2	–1	0	1	2	3	4

(iv) $-3 \leq x \leq -2,\ x \geq -1$