DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

2010

6 (a) Let $h(x) = x^2 + 1$, where $x \in \mathbf{R}$.
Write down a value of x for which $h(x) = 50$.

(b) Let $g(x) = \dfrac{1}{x-2}$, where $x \in \mathbf{R}$ and $x \neq 2$.

(i) Copy and complete the following table:

x	0	1	1·5	1·75	2·25	2·5	3	4
$g(x)$		–1		–4		2		

(ii) Draw the graph of the function g in the domain $0 \leq x \leq 4$.

(c) Let $f(x) = x - \dfrac{5}{x}$, where $x \in \mathbf{R}$ and $x \neq 0$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the co-ordinates of the two points at which the tangent to the curve is parallel to the line

ANSWERS

6 (a) $x = 7$ or $x = -7$

(b) (i)

x	0	1	1·5	1·75	2·25	2·5	3	4
$g(x)$	–0.5	–1	–2	–4	4	2	1	0.5

(c) (i) $f'(x) = 1 + \dfrac{5}{x^2}$ (ii) (1, –4), (–1, 4)

7 (a) Differentiate $x^2 - 6x + 1$ with respect to x.

(b) (i) Differentiate $5 - 3x$ with respect to x from first principles.

(ii) Given that $y = (x^2 - 4)(3x - 1)$, find the value of $\frac{dy}{dx}$ when $x = 2$.

(c) The speed, v, of an object at time t is given by

$$v = 96 + 40t - 4t^2$$

where t is in seconds and v is in metres per second.

(i) At what times will the speed of the object be 96 metres per second?

(ii) What will the acceleration of the object be at $t = 2{\cdot}5$ seconds?

(iii) At what value of t will the acceleration become negative?

8. Let $f(x) = x^3 - 3x + 1$, where $x \in \mathbf{R}$.

(i) Find $f(-3)$, $f(-2)$, $f(0)$, $f(2)$ and $f(3)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iv) Draw the graph of the function f in the domain $-3 \le x \le 3$.

(v) Find the range of values of k for which the equation

$$x^3 - 3x + 1 = k$$

has three real solutions (roots).

Answers

7 (a) $2x - 6$

(b) (i) -3 (ii) 20

(c) (i) $t = 0$ s, 10 s (ii) 20 metres per second squared

(iii) $t > 5$ s

8 (i) $f(-3) = -17$, $f(-2) = -1$, $f(0) = 1$, $f(2) = 3$, $f(3) = 19$

(ii) $f'(x) = 3x^2 - 3$

(iii) Local maximum $(-1, 3)$, Local minimum $(1, -1)$

(v) 3 solutions: $-1 < k < 3$