DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

2009

6 (a) Let $g(x) = 4 - kx$.
Given that $g(-5) = 34$, find the value of k.

(b) Let $h(x) = x(1 - x^2)$, where $x \in \mathbf{R}$.

(i) Verify that $h(3) + h(-3) = 0$.

(ii) Find the values of x for which $h'(x) = -11$, where $h'(x)$ is the derivative of $h(x)$.

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(c) Let $f(x) = x^3 - 6x^2 + 9x - 3$, where $x \in \mathbf{R}$.

(i) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(ii) Draw the graph of the function f in the domain $0 \le x \le 4$.

(iii) Use your graph to estimate the range of values of x for which $x < 3$ and $f(x) \ge 0$.

ANSWERS

6 (a) $k = 6$

(b) (ii) $x = \pm 2$

(c) (i) Local maximum (1, 1), Local Minimum (3, –3)

(iii) $0.5 \le x \le 1.6$

7 (a) Differentiate $3x^5 - 7x^2 + 9x$ with respect to x.

(b) (i) Given that $y = (x^2 - 4x)^5$, find the value of $\frac{dy}{dx}$ when $x = 2$.

(ii) Differentiate $\frac{x^2 - 1}{x^2 + 1}$ with respect to x.

Write your answer in the form $\frac{kx}{(x^2 + 1)^n}$, where $k, n \in \mathbf{N}$.

(c) A ball is fired straight up in the air.
The height, h metres, of the ball above the ground is given by

$$h = 30t - 5t^2$$

where t is the time in seconds after the ball was fired.

(i) After how many seconds does the ball hit the ground?

(ii) Find the speed of the ball after 2 seconds.

(iii) Find the maximum height reached by the ball.

ANSWERS

7 (a) $15x^4 - 14x + 9$

(b) (i) 0 (ii) $\frac{4x}{(x^2 + 1)^2}$

(c) (i) 6 s (ii) 10 m/s (iii) 45 m

8 (a) Let $g(x) = 2(6 - 3x)$, where $x \in \mathbf{R}$.
Find the value of x for which $g(x) = 0$.

(b) Differentiate $2x^2 - 5x$ with respect to x from first principles.

(c) Let $f(x) = \dfrac{1}{x+1}$, $x \in \mathbf{R}$, $x \neq -1$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the two values of x at which the slope of the tangent to the curve $y = f(x)$ is -1.

(iii) One of these tangents intersects the positive y-axis.
Find the equation of this tangent.

8 (a) $x = 2$

(b) $\dfrac{dy}{dx} = 4x - 5$

(c) (i) $-\dfrac{1}{(x+1)^2}$ (ii) $x = -2,\ 0$ (iii) $x + y - 1 = 0$