Differentiation & Functions (Q 6, 7 & 8, Paper 1)

2008

6 (a) Let $g(x) = 2x - 5$, where $x \in \mathbf{R}$.
Find the value of x for $g(x) = 19$.

(b) Differentiate $3x^2 + 5$ with respect to x from first principles.

(c) Let $f(x) = \dfrac{x^2 - x}{1 - x^3}$, $x \in \mathbf{R}$, $x \neq 1$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Show that the tangent to the curve $y = f(x)$ at the point (0, 0) makes an angle of 135° with the positive sense of the x-axis.

7 (a) Differentiate with respect to x
(i) x^7

(ii) $5x - 3x^4$.

(b) (i) Differentiate $(1 + 3x)(4 - x^2)$ with repsect to x.

(ii) Given that $y = (3x^2 - 4x)^8$, find $\dfrac{dy}{dx}$ when $x = 1$.

(c) A distress flare is tested by firing it vertically upwards from the top of a tower.
The height, h metres, of the flare above the ground is given by

$$h = 20 + 90t - 5t^2$$

where t is the time in seconds from the instant the flare is fired.
The flare is designed to explode 7 seconds after firing.
(i) Find the height above the ground at which the flare explodes.

(ii) Find the speed of the flare at the instant it explodes.

(iii) If the flare failed to explode, find the greatest height above the ground it would reach before falling back down.

Answers

6 (a) 12

(c) (i) $f'(x) = \dfrac{x^2 - 1}{(1 + x + x^2)^2}$

7 (a) (i) $7x^6$ (ii) $5 - 12x^3$

(b) (i) $12 - 2x - 9x^2$ (ii) -16

(c) (i) 405 m (ii) 20 m/s (iii) 425 m

8 Let $f(x) = x^3 - 9x^2 + 24x - 18,$ where $x \in \mathbf{R}$.

(i) Find $f(1)$ and $f(5)$.

(ii) Find $f'(x),$ the derivative of $f(x)$.

(iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iv) Draw the graph of the function f in the domain $1 \le x \le 5$.

(v) Use your graph to write down the range of values of x for which $f'(x) < 0$.

(vi) The line $y = -3x + c$ is a tangent to the curve $y = f(x)$. Find the value of c.

ANSWERS

8 (i) $-2,\ 2$

(ii) $f'(x) = 3x^2 - 18x + 24$

(iii) Local maximum: (2, 2); Local minimum: (4, –2)

(v) $2 < x < 4$

(vi) $c = 9$