## Differentiation \& Functions (Q 6, 7 \& 8, Paper 1)

2007

6 (a) Let $g(x)=x^{2}-6 x, x \in \mathbf{R}$.
(i) Write down $g^{\prime}(x)$, the derivative of $g(x)$.
(ii) For what value of $x$ is $g^{\prime}(x)=0$ ?
(b) A cold object is placed in a warm room.

Its temperature $C$ degrees after time $t$ minutes is shown in the following graph.

(i) After what time interval is the temperature of the object 0 degrees?
(ii) What is the rise in temperature of the object in the first 10 minutes?
(iii) The relationship between the temperature $C$ and the time $t$ is given by

$$
C=\frac{1}{2}(t+k) .
$$

Find the value of $k$.
(c) Let $f(x)=(5 x-2)^{4}$ for $x \in \mathbf{R}$.
(i) Find $f^{\prime}(x)$, the derivative of $f(x)$.
(ii) Find the co-ordinates of the point on the curve $y=f(x)$ at which the slope of the tangent is 20 .

## Answers

6
(a) (i) $g^{\prime}(x)=2 x-6$
(ii) $x=3$
(b) (i) 6 minutes
(ii) 5 degrees
(iii) $k=-6$
(c) (i) $f^{\prime}(x)=20(5 x-2)^{3}$
(ii) $\left(\frac{3}{5}, 1\right)$

7 (a) Differentiate $6 x^{4}-3 x^{2}+7 x$ with respect to $x$.
(b) (i) Differentiate $\left(x^{2}+9\right)\left(4 x^{3}+5\right)$ with respect to $x$.
(ii) Given that $y=\frac{3 x}{2 x+3}$, find $\frac{d y}{d x}$.

Write your answer in the form $\frac{k}{(2 x+3)^{n}}$, where $k, n \in \mathbf{N}$.
(c) A car starts from rest at the point $a$.


The distance of the car from $a$, after $t$ seconds, is given by

$$
s=2 t^{2}+2 t
$$

where $s$ is in metres.
(i) Find the speed of the car after 2 seconds.
(ii) Find the acceleration of the car.
(iii)The distance from $a$ to the point $b$ is 24 metres. After how many seconds does the car reach the point $b$ ?

8 (a) Let $f(x)=\frac{1}{4}(6-2 x)$ for $x \in \mathbf{R}$. Evaluate $f(5)$.
(b) Differentiate $x^{2}-3 x$ with respect to $x$ from first principles.
(c) Let $f(x)=\frac{1}{x+7}, x \in \mathbf{R}, x \neq-7$.
(i) Given that $f(k)=1$, find $k$.
(ii) Find $f^{\prime}(x)$, the derivative of $f(x)$.
(iii) Show that the curve $y=f(x)$ has no turning points.

## Answers

7 (a) $24 x^{3}-6 x+7$
(b) (i) $20 x^{4}+108 x^{2}+10 x$ (ii) $\frac{9}{(2 x+3)^{2}}$
(c) (i) $10 \mathrm{~ms}^{-1}$
(ii) $4 \mathrm{~ms}^{-2}$
(iii) 3 s

8 (a) $f(5)=-1$
(b) $2 x-3$
(c) (i) $k=-6$
(ii) $f^{\prime}(x)=-\frac{1}{(x+7)^{2}}$

