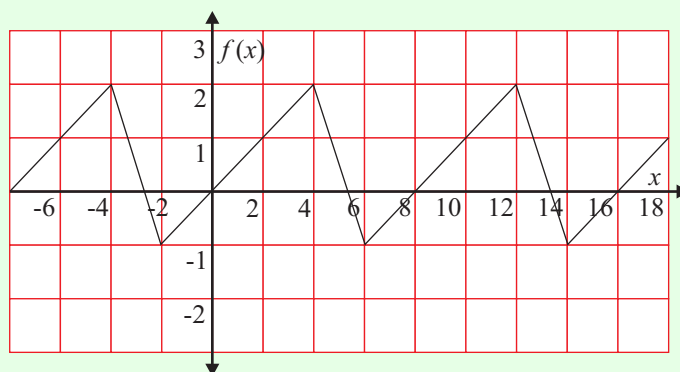


DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

2006

- 6 (a) $f : x \rightarrow f(x)$ is a periodic function defined for $x \in \mathbf{R}$.
The period is as indicated in the diagram.



- (i) Write down the period and the range of the function.
 - (ii) Find $f(44)$.
- (b) The temperature, C , in degrees Celsius, of a liquid in an insulated container is related to time t , in hours, by
- $$C = 86 - 6t.$$
- (i) Draw the straight line graph of this relation, putting t on the horizontal axis, for $0 \leq t \leq 8$.
 - (ii) Use your graph to estimate the temperature when $t = 5.5$ hours.
 - (iii) Use your graph to estimate the time it takes for the temperature to fall from 80 degrees to 60 degrees.
- (c) Let $f(x) = 3 + 8x - 2x^2$, $x \in \mathbf{R}$.
- (i) Find the co-ordinates of the point at which the curve $y = f(x)$ cuts the y -axis.
 - (ii) Find the value of x for which $f(x)$ is a maximum.
 - (iii) For what range of values of x is $f'(x) > 4$?

ANSWERS

- | | | |
|------------------------|-----------------|---------------|
| 6 (a) (i) 8, $[-1, 2]$ | (ii) 2 | |
| (b) (ii) 53 degrees | (iii) 3.3 hours | |
| (c) (0, 3) | (ii) $x = 2$ | (iii) $x < 1$ |

7 (a) Differentiate $5x^3 - 4x + 7$ with respect to x .

(b) (i) Differentiate $\frac{x^2 - 1}{x + 1}$ with respect to x and write your answer in its simplest form.

(ii) Given that $y = (5 - x^2)^3$, find $\frac{dy}{dx}$ when $x = 2$.

(c) A missile is fired straight up in the air. The height, h metres, of the missile above the firing position is given by

$$h = t(200 - 5t)$$

where t is the time in seconds from the instant the missile was fired.

(i) Find the speed of the missile after 10 seconds.

(ii) Find the acceleration of the missile.

(iii) One second before reaching its greatest possible height, the missile strikes a target. Find the height of the target.

8 (a) Let $g(x) = \frac{3}{x+1}$, $x \in \mathbf{R}$, $x \neq -1$.

Evaluate $g(0.5) - g(-0.5)$.

(b) Let $h(x) = x^2 + 2x - 1$, $x \in \mathbf{R}$.

(i) Simplify $h(x-5)$.

(ii) Find the value of x for which $h(x-5) = h(x) - 5$.

(c) Let $f(x) = \frac{1}{x-2}$, $x \in \mathbf{R}$, $x \neq 2$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the values of x for which $f'(x) = -1$.

(iii) Find the co-ordinates of the two points on the curve $y = f(x)$ at which the slope of the tangent is -1 .

ANSWERS

7 (a) $15x^2 - 4$

(b) (i) 1

(ii) -12

(c) (i) 100 ms^{-1}

(ii) -10 ms^{-2}

(iii) 1995 m

8 (a) -4

(b) (i) $x^2 - 8x + 14$

(ii) $x = 2$

(c) (i) $-\frac{1}{(x-2)^2}$

(ii) $x = 1, 3$

(iii) $(1, -1), (3, 1)$