DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

2004

6 (a) Let $g(x) = 1 - kx$.

Given that $g(-3) = 13$, find the value of k.

(b) Let $f(x) = x^3 - 3x^2 + 1$, $x \in \mathbf{R}$.

(i) Find $f(-1)$ and $f(3)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iv) Draw the graph of the function f in the domain $-1 \le x \le 3$.

Use your graph to:

(v) estimate the range of values of x for which $f(x) < 0$ and $x > 0$

(vi) estimate the range of values of x for which $f'(x) < 0$.

ANSWERS

6 (a) 4

(b) (i) $-3, 1$ (ii) $3x^2 - 6x$ (iii) $(0, 1), (2, -3)$

(v) $0.7 < x < 2.7$ (vi) $0 < x < 2$

7 (a) Differentiate with respect to x:

(i) $2x^5$

(ii) $4(3-x^2)$.

(b) (i) Differentiate $(x^2-4)(x^2+3x)$ with respect to x.

(ii) Given that $y=(x^2-2x-3)^3$, show that $\dfrac{dy}{dx}=0$ when $x = 1$.

(c) A jet is moving along an airport runway. At the instant it passes a marker it begins to accelerate for take-off. From the time the jet passes the marker, its distance from the marker is given by

$$s=2t^2+3t,$$

where s is in metres and t is in seconds.

(i) Find the speed of the jet at the instant it passes the marker ($t = 0$).

(ii) The jet has to reach a speed of 83 metres per second to take off. After how many seconds will the jet reach this speed?

(iii) How far is the jet from the marker at that time?

(iv) Find the acceleration of the jet.

8 (a) Let $g(x)=3x-7$.

(i) Find $g(7)$.

(ii) Find the value of k for which $g(7) = k[g(0)]$.

(b) Differentiate x^2+3x with respect to x from first principles.

(c) Let $f(x)=\dfrac{1}{x+3}$, $x\in\mathbf{R}$, $x\neq -3$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) There are two points on the curve $y = f(x)$ at which the slope of the tangent is -1. Find the co-ordinates of these two points.

(iii) Show that no tangent to the curve $y = f(x)$ has a slope of 1.

ANSWERS

7 (a) (i) $10x^4$ (ii) $-8x$

(b) (i) $4x^3+9x^2-8x-12$ (ii) $(6x-6)(x^2-2x-3)^2$

(c) (i) $3\ \text{m s}^{-1}$ (ii) 20 s (iii) 860 m (iv) $4\ \text{m s}^{-2}$

8 (a) (i) 14 (ii) -2

(b) $2x + 3$

(c) (i) $-\dfrac{1}{(x+3)^2}$ (ii) $(-2, 1)$, $(-4, -1)$