## Differentiation \& Functions (Q 6, 7 \& 8, Paper 1)

2003
6 (a) Let $g(x)=\frac{2 x}{3}-1$.
Find the value of $x$ for which $g(x)=5$.
(b) Differentiate $x^{2}-2 x$ with respect to $x$ from first principles.
(c) Let $f(x)=3-5 x-2 x^{2}, x \in \mathbf{R}$.
(i) Find $f^{\prime}(x)$, the derivative of $f(x)$, and hence find the co-ordinates of the local maximum point of the curve $y=f(x)$.
(ii) Solve the equation $f(x)=0$.
(iii) Use your answers from parts (i) and (ii) to sketch the graph of $f: x \rightarrow 3-5 x-2 x^{2}$, showing scaled and labelled axes.

7 (a) Differentiate with respect to $x$ :
(i) $x^{3}$
(ii) $\frac{x^{2}-x^{4}}{2}$.
(b) (i) Differentiate $\left(3 x^{3}-2 x^{2}+2\right)^{4}$ with respect to $x$.
(ii) Given that $y=\left(5 x^{2}+3\right)\left(4-x^{2}\right)$, find $\frac{d y}{d x}$ when $x=1$.
(c) A train is travelling along a track. Suddenly, the brakes are applied. From the time the brakes are applied ( $t=0$ seconds), the distance travelled by the train, in metres, is given by

$$
s=30 t-\frac{1}{4} t^{2} .
$$

(i) What is speed of the train at the moment the brakes are applied?
(ii) How many seconds does it take for the train to come to rest?
(iii) How far does the train travel in that time?

Answers
6 (a) 9
(b) $2 x-2$
(c) (i) $f^{\prime}(x)=-5-4 x ;\left(-\frac{5}{4}, \frac{49}{8}\right)$
(ii) $-3, \frac{1}{2}$

7
(a) (i) $3 x^{2}$
(ii) $x-2 x^{3}$
(b) (i) $\left(36 x^{2}-16 x\right)\left(3 x^{3}-2 x^{2}+2\right)^{3}$
(ii) 14
(c) (i) $30 \mathrm{~ms}^{-1}$
(ii) 60 s
(iii) 900 m

8 (a) Part of the graph of a periodic function is shown.
Write down the period and range of the function.

(b) (i) The function $g$ is defined for natural numbers by the rule:

$$
g(x)=\left\{\begin{array}{l}
0 \text { if is even. } \\
1 \text { if is odd }
\end{array}\right.
$$

Find $g(13)+g(14)+g(15)$.
(ii) Given that $h(x)=x^{2}$, write down $h(x+3)$.

Hence, find the value of $x$ for which $h(x)=h(x+3)$.
(c) Let $f(x)=x^{3}+2 x^{2}-1$.
(i) Find $f^{\prime}(x)$, the derivative of $f(x)$.
(ii) $L$ is the tangent to the curve $y=f(x)$ at $x=-\frac{2}{3}$.

Find the slope of $L$.
(iii) Find the two values of $x$ at which the tangents to the curve $y=f(x)$ are perpendicular to $L$.

## Answers

8 (a) 4; [0, 3]
(b) (i) 2
(ii) $x^{2}+6 x+9 ;-\frac{3}{2}$
(c) (i) $3 x^{2}+4 x$
(ii) $-\frac{4}{3}$
(iii) $-\frac{3}{2}, \frac{1}{6}$

