## DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

## 2001

6 (a) Let 
$$g(x) = \frac{1}{x^2 + 1}$$
 for  $x \in \mathbf{R}$ .

Evaluate

(i) *g*(2)

- (ii) g(3) and write your answers as decimals.
- (b) Let  $f(x) = 2 9x + 6x^2 x^3$  for  $x \in \mathbf{R}$ .
  - (i) Find f(-1), f(2) and f(5).
  - (ii) Find f'(x), the derivative of f(x).
  - (iii) Find the co-ordinates of the local maximum and the local minimum of f(x).
  - (iv) Draw the graph of f(x) in the domain  $-1 \le x \le 5$ .
  - (v) Use your graph to find the range of real values of k for which f(x) = k has more than one solution.
- 7 (a) Differentiate with respect to x

(i) 
$$6x^5 + x^2$$

(ii) 
$$(x-3)(x+3)$$

(b) (i) Find 
$$\frac{dy}{dx}$$
 when  $y = \frac{x^2}{x-4}$ ,  $x \neq 4$ .

(ii) Find the value of 
$$\frac{dy}{dx}$$
 at  $x = 0$  when  $y = (x^2 - 7x + 1)^5$ .

(c) Two fireworks were fired straight up in the air at t = 0 seconds.The height, h metres, which each firework reached above the ground t seconds after it was fired is given by

$$h = 80t - 5t^2$$
.

The first firework exploded 5 seconds after it was fired.

(i) At what height was the first firework when it exploded?

(ii) At what speed was the first firework travelling when it exploded?

The second firework failed to explode and it fell back to the ground.

(iii) After how many seconds did the second firework reach its maximum height?

| Answers |                                   |                            |                       |                      |
|---------|-----------------------------------|----------------------------|-----------------------|----------------------|
| 6       | (a)(i) 0.2                        | (ii) 0.1                   |                       |                      |
|         | (b)(i) 18, 0, -13                 | (ii) $-9+12x-3x^2$         | (iii) (1, -2), (3, 2) | (v) $-2 \le k \le 2$ |
| 7       | (a)(i) $30x^4 + 2x$               | (ii) 2 <i>x</i>            |                       |                      |
|         | (b)(i) $\frac{x^2 - 8x}{(x-4)^2}$ | (ii) -35                   |                       |                      |
|         | (c)(i) 275 m                      | (ii) $30 \text{ m s}^{-1}$ | (iii) 8 s             |                      |

- 8 (a) Let  $g(x) = x^4 32x$  for  $x \in \mathbf{R}$ .
  - (i) Write down g'(x), the derivative of g(x).
  - (ii) For what value of x is g'(x) = 0?
  - (b) Differentiate  $3x^2 x$  from first principles with respect to x.
  - (c) Let  $f(x) = \frac{1}{x+1}$  for  $x \in \mathbf{R}$  and x > -1.

(i) Find 
$$f'(x)$$
.

- (ii) Find the co-ordinates of the point on the curve of f(x) at which the tangent has slope of  $-\frac{1}{4}$ .
- (iii) Find the equation of the tangent to the curve which has slope of  $-\frac{1}{4}$ .

| Answers |                              |                         |  |
|---------|------------------------------|-------------------------|--|
| 8       | (a) (i) $4x^3 - 32$          | (ii) 2                  |  |
|         | (b) $6x - 1$                 |                         |  |
|         | (c) (i) $-\frac{1}{(x+1)^2}$ | (ii) $(1, \frac{1}{2})$ |  |
|         | (iii) $x + 4y - 3 = 0$       |                         |  |