## Differentiation \& Functions (Q 6, 7 \& 8, Paper 1)

## 2000

6 (a) Differentiate $7 x+3$ from first principles with respect to $x$.
(b)


The graph shows portion of a periodic function $f: x \rightarrow f(x)$ which is defined for $x \in \mathbf{R}$.
(i) Write down the period and the range of $f(x)$.
(ii) Complete the following table:

| $x$ | 2 | 8 | 14 | 20 | 26 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ |  |  |  |  |  |

(c) Let $g(x)=(2 x+3)\left(x^{2}-1\right)$ for $x \in \mathbf{R}$.
(i) For what two values of $x$ is the slope of the tangent to the curve of $g(x)$ equal to 10 ?
(ii) Find the equations of the two tangents to the curve of $g(x)$ which have slope 10.

## Answers

6 (a) 7
(b) (i) $8,[0,10]$

(ii) | $x$ | 2 | 8 | 14 | 20 | 26 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 10 | 0 | 5 | 0 | 0 |

(c)
(i) $-2,1$
(ii) $10 x-y+17=0,10 x-y-10=0$

7 (a) Differentiate with respect to $x$
(i) $4 x^{2}+5$
(ii) $9 x-x^{3}$.
(b) (i) Find $\frac{d y}{d x}$ when $y=\frac{2 x-7}{x-1}, x \neq 1$.
(ii) Find $\frac{d y}{d x}$ when $y=\left(x^{2}+5 x-1\right)^{3}$.
(c) A car, starting at $t=0$ seconds, travels a distance of $s$ metres in $t$ seconds where

$$
s=30 t-\frac{9}{4} t^{2}
$$

(i) Find the speed of the car after 2 seconds.
(ii) After how many seconds is the speed of the car equal to zero?
(iii) Find the distance travelled by the car up to the time its speed is zero.

8 (a) Let $p(x)=3 x-12$.
For what values of $x$ is $p(x)<0$ where $x$ is a positive whole number?
(b) (i) Draw the graph of

$$
g(x)=\frac{1}{x} \text { for }-3 \leq x \leq 3, x \in \mathbf{R} \text { and } x \neq 0 .
$$

(ii) Using the same axes and the same scales, draw the graph of

$$
h(x)=x+1 \text { for }-3 \leq x \leq 3, x \in \mathbf{R} .
$$

(iii) Use your graphs to estimate the values of $x$ for which

$$
\frac{1}{x}=x+1 .
$$

(c) Let $f(x)=x^{3}-3 x^{2}+a x+1$ for all $x \in \mathbf{R}$ and for $a \in \mathbf{R}$.
$f(x)$ has a turning point (a local maximum or a local minimum) at $x=-1$.
(i) Find the value of $a$.
(ii) Is this turning point a local maximum or a local minimum?

Give a reason for your answer.
(iii) Find the co-ordinates of the other turning point of $f(x)$.

## Answers

7 (a) (i) $8 x$
(ii) $9-3 x^{2}$
(b) (i) $\frac{5}{(x-1)^{2}}$
(ii) $(6 x+15)\left(x^{2}+5 x-1\right)^{2}$
(c) (i) $21 \mathrm{~ms}^{-1}$
(ii) $\frac{20}{3} \mathrm{~s}$
(iii) 100 m

8 (a) $\{1,2,3\}$
(b) (iii) $-1.6<x<0.6$
(c) (i) $a=-9$
(ii) $\frac{d^{2} y}{d x^{2}}=-12$
(iii) $(3,-26)$

