DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

1999

6 (a) Let $f(x) = 2(3x-1),\ x \in \mathbf{R}$.
Find the value of x for which $f(x) = 0$.

(b) Differentiate from first principles

$$x^2 + 5x$$

with respect to x.

(c) Let $f(x) = x^3 - 6x^2 + 12$ for $x \in \mathbf{R}$.
Find the derivative of $f(x)$.
At the two points (x_1, y_1) and (x_2, y_2), the tangents to the curve $y = f(x)$ are parallel to the x axis, where $x_2 > x_1$.
Show that

(i) $x_2 - x_1 = 4$

(ii) $y_2 = y_1 - 32$.

7 (a) Differentiate

$$2x^3 - 7$$

with respect to x.

(b) (i) Find $\frac{dy}{dx}$ when $y = (3-7x)^5$.

(ii) Find $\frac{dy}{dx}$ when $y = \frac{x^2}{1-x}$, $x \neq 1$. Show that $\frac{dy}{dx} = 0$ at $x = 0$.

(c) The speed, v, in metres per second, of a body after t seconds is given by

$$v = 3t(4-t).$$

(i) Find the acceleration at each of the two instants when the speed is 9 metres per second.

(ii) Find the speed at the instant when the acceleration is zero.

ANSWERS

6 (a) $\frac{1}{3}$

(b) $2x + 5$

(c) $3x^2 - 12x;\ x_1 = 0,\ x_2 = 4;\ y_1 = 12,\ y_2 = -20$

7 (a) $6x^2$

(b) (i) $-35(3-7x)^4$ (ii) $\frac{2x - x^2}{(1-x)^2}$

(c) (i) $6\ \text{ms}^{-2}, -6\ \text{ms}^{-2}$ (ii) $12\ \text{ms}^{-1}$

8 Let $f(x) = 2x^3 - 5x^2 - 4x + 3$ for $x \in \mathbf{R}$.

(i) Complete the table

x	–1.5	–1	0	1	2	3	3.5
$f(x)$	–9						13.5

(ii) Find the derivative of $f(x)$.
Calculate the co-ordinates of the local minimum and show that the co-ordinates of the local maximum are $(-\frac{1}{3}, \frac{100}{27})$.

(iii) Draw the graph of

$$f(x) = 2x^3 - 5x^2 - 4x + 3$$

for $-1.5 \le x \le 3.5$.

(iv) Write the equation $2x^3 - 5x^2 - 6x + 6 = 0$ in the form

$$2x^3 - 5x^2 - 4x + 3 = ax + b, \quad a, b \in \mathbf{Z}.$$

Hence, use your graph to estimate the solutions of the equation

$$2x^3 - 5x^2 - 6x + 6 = 0.$$

ANSWERS

8 (i)

x	–1.5	–1	0	1	2	3	3.5
$f(x)$	–9	0	3	–4	–9	0	13.5

(ii) $6x^2 - 10x - 4$; $(2, -9)$

(iv) $a = 2$, $b = -3$; $x = -1.4, 0.7, 3.2$