1998

6 (a) If $f(x) = 5x - 8$ and $g(x) = 13 - 2x$, find the value of x for which

$$f(x) = g(x).$$

(b) The speed, v, in metres per second of an engine moving along a track is related to time, t, in seconds by

$$v = \tfrac{1}{3}(2t + 5).$$

(i) Draw the straight line graph of this relation, putting t on the horizontal axis, for $0 \leq t \leq 8$.

(ii) Use your graph to estimate the speed when $t = 2.5$ seconds.

(iii) Use your graph to estimate the time at which the speed reaches 6 metres per second.

(c) $f(x) = (x + k)(x - 2)^2$, where k is a real number.

(i) If $f(3) = 7$, find the value of k.

(ii) Using this value for k, find the coordinates of the local maximum and of the local minimum of $f(x)$.

ANSWERS

6 (a) 3

(b) (ii) $3.3\ \mathrm{m\,s^{-1}}$ (iii) 6.5 s

(c) (i) 4 (ii) (2, 0), (–2, 32)

7 (a) Differentiate with respect to x

(i) $x^2 - 3x$

(ii) $\frac{1}{x^2}$.

(b) (i) Find $\frac{dy}{dx}$ when $y = \frac{2x}{x^2+1}$.

(ii) Find $\frac{dy}{dx}$ when $y = (4-3x^2)^7$ and write down the range of values of x for which $\frac{dy}{dx} > 0$.

(c) The volume of water, V, in cm^3, that remains in a leaking tank after t seconds is given by

$$V = 45000 - 300t + 0.5t^2.$$

(i) After how many seconds will the tank be empty?

(ii) Find the rate of change of the volume with respect to t when $t = 50$ seconds.

8 Let $f(x) = \frac{1}{x-1}$, for $x \in \mathbf{R}$ and $x \neq 1$.

(i) Find the value of $f(-2)$, $f(0)$, $f(\frac{3}{2})$ and $f(5)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Draw the graph of $f(x)$ for $-2 \le x \le 5$.

(iv) Find the equation of the tangent T to the curve at the point $(0, -1)$.

(v) Find the coordinates of the other point on the graph of $f(x)$ at which the tangent to the curve is parallel to T.

Answers

7 (a) (i) $2x-3$ (ii) $-\frac{2}{x^3}$

(b) (i) $\frac{2-2x^2}{(x^2+1)^2}$ (ii) $-42x(4-3x^2)^6$, $x<0$

(c) (i) 300 s (ii) -250 cm^3/s

8 (i) $-\frac{1}{3}$, -1, 2, $\frac{1}{4}$ (ii) $-\frac{1}{(x-1)^2}$

(iv) $x + y + 1 = 0$ (v) (2, 1)