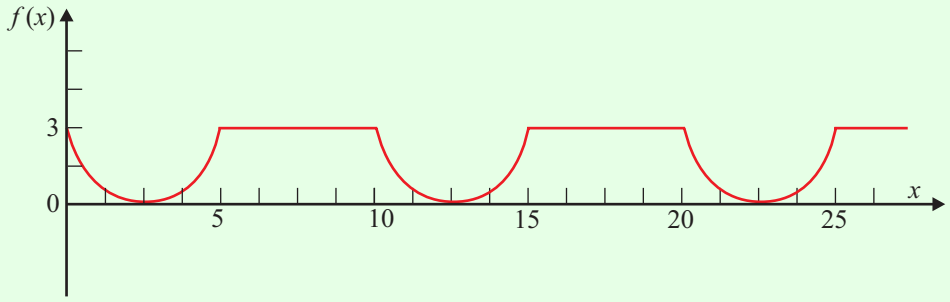
DIFFERENTIATION & FUNCTIONS (Q 6, 7 & 8, PAPER 1)

1997

6 (a)



The graph shows portion of a periodic function $f : x \to f(x)$.
Write down the period and range of the function.
What is the value of $f(77.5)$?

(b) Differentiate from first principles

$$3x^2 - 2$$

with respect to x.

(c) Let $f(x) = ax^3 + bx + c$, for all $x \in \mathbf{R}$ and for $a, b, c \in \mathbf{R}$.
Use the information which follows to find the value of a, of b and of c:
(i) $f(0) = 3$
(ii) the slope of the tangent to the curve of $f(x)$ at $x = 1$ is -18
(iii) the curve of $f(x)$ has a local maximum at $x = 2$.

ANSWERS

6 (a) 10, [0, 3]; 3
(b) $6x$
(c) $a = 2, b = -24, c = 3$

7 (a) Differentiate with respect to x

(i) $-x^2$

(ii) $x^4 + x + 1$.

(b) (i) Find $\frac{dy}{dx}$ when $y = (x^2 - 3)(1 - x)$.

(ii) Find the value of $\frac{dy}{dx}$ at $x = -1$ when $y = (3x + 1)^4$.

(c) The distance s metres of an object from a fixed point at t seconds is given by

$$s = \frac{t+1}{t+3}.$$

(i) At what time is the object 0.75 m from a fixed point?

(ii) What is the speed of the object, in terms of t, at t seconds?

(iii) After how many seconds will the speed of the object be less than 0.02 m/s?

8 (a) Let $f(x) = x^2 - 4x$, for $x \in \mathbf{R}$.

Find $f'(x)$, the derivative of $f(x)$.

For what value of x is $f'(x) = 0$?

(b) Find the equation of the tangent to the curve

$$y = x^3 - 4x + 7$$

at the point where $x = 1$.

(c) Draw a graph of

$$g(x) = \frac{1}{x+2}$$

for $0 \le x \le 4$, $x \in \mathbf{R}$.

Using the same axes and the same scales draw the graph of

$$h(x) = x - 2.$$

Show how your graphs may be used to estimate the value of $\sqrt{5}$.

ANSWERS

7 (a) (i) $-2x$ (ii) $4x^3 + 1$

(b) (i) $-3x^2 + 2x + 3$ (ii) -96

(c) (i) 5 seconds (ii) $\frac{2}{(t+3)^2}$ (iii) 7 seconds

8 (a) $2x - 4$, 2

(b) $x + y - 5 = 0$