1996

6 (a) Let $f(x) = 3x + k,\ x \in \mathbf{R}$.
If $f(5) = 0$, find the value of k.

(b) Let $g(x) = x^2 + bx + c,\ x \in \mathbf{R}$.
The solutions of $g(x) = 0$ are symmetrical about the line $x = 1$.
If $x = -3$ is one solution of $g(x) = 0$, find the other solution.
Find the value of b and the value of c.

(c) Let $f(x) = \dfrac{1}{x-2}$, for $x \in \mathbf{R}$ and $x \neq 2$.
Find the derivative of $f(x)$.
Tangents to $f(x)$ make an angle of 135° with the x axis.
Find the coordinates of the points on the curve of $f(x)$ at which this occurs.

ANSWERS

6 (a) -15
(b) $x = 5$; $-1 \le x \le 3$.
(c) $-\dfrac{1}{(x-2)^2}$; $(1, -1)$, $(3, 1)$

7 (a) Differentiate from first principles

$$3x-7$$

with respect to x.

(b) (i) Find $\frac{dy}{dx}$ when $y=\frac{2x}{4-x^2}$, for $x\in\mathbf{R}$ and $x\neq\pm 2$.

Show that $\frac{dy}{dx}>0$.

(ii) Differentiate $\left(x^5-\frac{1}{x^2}\right)^7$ with respect to x, $x\neq 0$.

(c) A stone is dropped from a height of 80 metres. Its height h metres above the ground after t seconds is given by

$$h=80-t^2.$$

Find
(i) its speed after t seconds
(ii) its speed after 2.5 seconds
(iii) the time it takes to fall the first 14.4 metres.

8 (a) Find $\frac{ds}{dt}$ when $s=6t^2-3t+7$.

(b) Let $f(x)=x^3-3x^2$, for $x\in\mathbf{R}$.

(i) Find $f'(x)$, the derivative of $f(x)$. Hence, calculate the coordinates of the local maximum and the local minimum of $f(x)$.

(ii) Draw the graph of

$$f(x)=x^3-3x^2$$

for $-1\leq x\leq 3$.

(iii) Use your graph to estimate the values of x for which

$$f(x)+2=0.$$

(iv) Use your graph to estimate the range of values of x for which

$$f'(x)<0.$$

Answers

7 (a) (a) 3

(b) (i) $-\frac{2x^2+4}{(4-x^2)^2}$ (ii) $\left(35x^4+\frac{14}{x^3}\right)\left(x^5-\frac{1}{x^2}\right)^6$

(c) (i) $2t\ \mathrm{m\,s^{-1}}$ (ii) $5\ \mathrm{m\,s^{-1}}$ (iii) 8.1 s

8 (a) $12t-3$

(b) (i) $3x^2-6x$; $(0,0)$, $(2,-4)$ (iii) $x=-0.7, 1, 2.7$

(iv) $0<x<2$