ARITHMETIC (Q 1, PAPER 1)

2005

1 (a) Express 35 cm as a fraction of 1 m. Give your answer in its simplest form.

   (b) (i) The approximation 50×80 was used for the calculation 51×79.
            Find the percentage error, correct to one decimal place.

        (ii) Express the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ as a ratio of natural numbers.

            Divide 325 in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

   (c) At the start of the year 2000 the population of a particular town was P.
       During the year 2000, the population of the town increased by 10%.
       (i) Express, in terms of P, the population of the town at the end of the year 2000.

        (ii) During the year 2001 the population of the town increased by 4%.
            During the year 2002 the population increased by 2%.
            Find the total percentage increase in the population of the town over the three years.

        (iii) The actual increase in the population was 8344. Find the value of P.

SOLUTION

1 (a) Express each quantity in the same units, say cm.

1 kilometre (km) = 1000 m
100 centimetres (cm) = 1 m
1000 millimetres (mm) = 1m

$$\therefore \frac{35 \text{ cm}}{100 \text{ cm}} = \frac{7}{20}$$

1 (b) (i) FINDING THE PERCENTAGE ERROR

> **STEPS**
>
> **1**. Find the absolute error: Absolute error $= |\text{True value} - \text{Estimate}|$
>
> **2**. Find the fractional error: Fractional error $= \dfrac{\text{Absolute Error}}{\text{True Value}}$
>
> **3**. Find the percentage error: % Error $= \dfrac{\text{Absolute Error}}{\text{True Value}} \times 100\%$

1. True value: $51 \times 79 = 4029$ Estimate: $50 \times 80 = 4000$

   Absolute Error $= |4029 - 4000| = 29$

2. Fractional error $= \dfrac{29}{4029}$

3. % error $= \dfrac{29}{4029} \times 100\% = 0.7\%$

$$\% \text{ Error } = \frac{\text{Absolute Error}}{\text{True Value}} \times 100\%$$

.... **2**

1 (b) (ii) Multiply each fraction by the lowest common denominator which is 12.

$$\therefore \frac{1}{2}:\frac{1}{3}:\frac{1}{4}=6:4:3$$

Add the three numbers: $6 + 4 + 3 = 13$

First number: $\frac{6}{13}\times 325 = 150$

Second number: $\frac{4}{13}\times 325 = 100$

Third number: $\frac{3}{13}\times 325 = 75$

1 (c)

(i) Multiply P by 1.1.
Population at end of 2000 $= 1.1P$

(ii) Population after 3 years $= 1.1P\times 1.04\times 1.02 = 1.16688P$
To find the percentage increase from this result, follow the steps to increase a quantity by a percentage backwards.
$1.16688 - 1 = 0.16688 = 16.688\%$

(iii) % increase $= 16.688\% = 0.16688$

$$\therefore P = \frac{8344}{0.16688} = 50,000$$