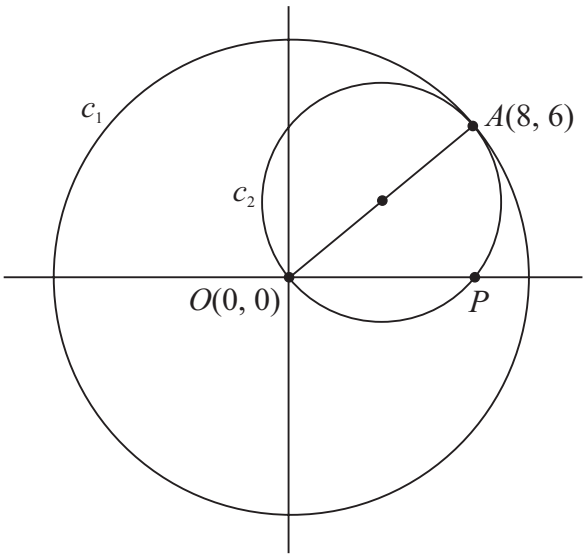
LC 2013 (Set D): Paper 2

Question 4 (25 marks)

Question 4 (a)



Find the radius r_1 of c_1 by finding the distance $|OA|$.

$O(0, 0) = (x_1, y_1),\ A(8, 6) = (x_2, y_2)$

$$|OA| = r = \sqrt{(8-0)^2 + (6-0)^2} = \sqrt{64+36} = \sqrt{100} = 10$$

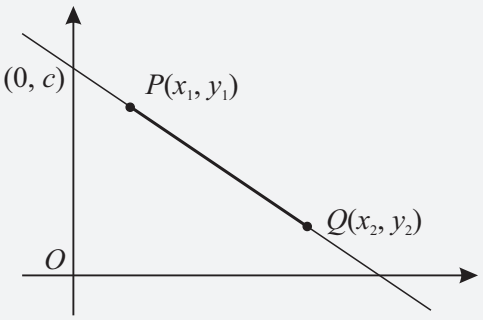
Equation of c_1:

Centre $(0, 0)$, $r_1 = 10$ $\quad \boxed{x^2 + y^2 = r^2}$ Equation of circle with centre (0, 0).

$$x^2 + y^2 = 10^2$$

$$x^2 + y^2 = 100$$

Formulae and Tables Book
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Length of PQ [page 18]

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of PQ [page 18]

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

Marking Scheme Notes
Question 4 (a) [Scale 5C (0, 3, 4, 5)]

3: • Writes a correct distance formula or formula for the equation of the circle

4: • Calculates the distance O to A
• Substitutes into correct circle formula with one error
• Calculates r^2 incorrectly
Note: A correct answer without work shown, award full credit

Question 4 (b)

Equation of c_2:

The centre of c_2 is the midpoint of OA.

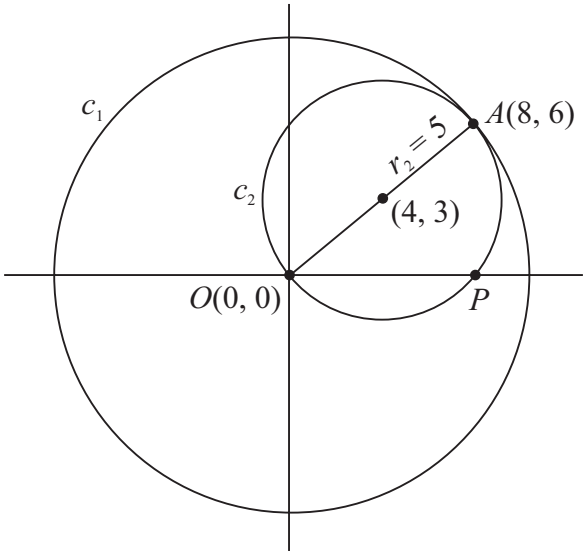
$O(0, 0) = (x_1, y_1),\ A(8, 6) = (x_2, y_2)$

$$\text{Midpoint} = \left(\frac{0+8}{2}, \frac{0+6}{2}\right) = (4, 3)$$

$$\text{Radius } r_2 = \tfrac{1}{2}|OA| = 5$$

Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r
$$(x-h)^2+(y-k)^2=r^2$$

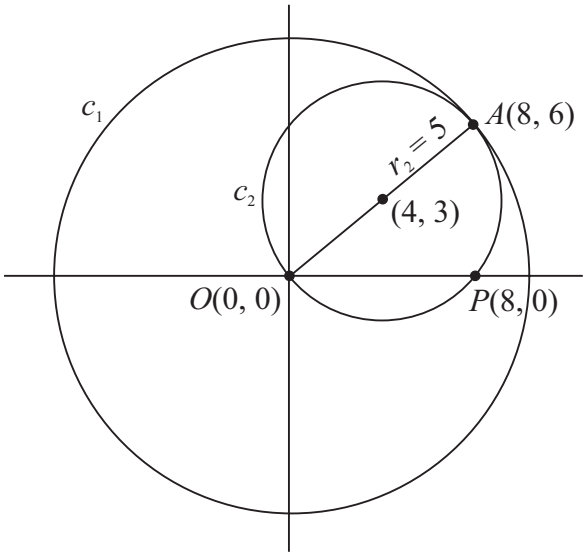
Centre $(h, k) = (4, 3)$, $r = 5$
$c_2 : (x-4)^2+(y-3)^2=5^2$
$(x-4)^2+(y-3)^2=25$



Marking Scheme Notes
Question 4 (b) [Scale 10D (0, 2, 5, 8, 10)]
2: • Writes a correct midpoint formula or formula for the equation of the circle
• Writes the midpoint or radius correctly
5: • Writes the midpoint and radius correctly
• Substitutes the centre or radius into the equation.
8: • Substitutes the centre and radius into the equation
• Substitutes into correct circle formula with one error
• Calculates r^2 incorrectly

Question 4 (c)

$c_2 : (x-4)^2+(y-3)^2=25$
Cuts x-axis: Put $y = 0$
$\therefore (x-4)^2+(0-3)^2=25$
$(x-4)^2+9=25$
$(x-4)^2=25-9=16$
$x-4=\pm\sqrt{16}=\pm4$
$\therefore x = 0, 8$
Answer: $P(8, 0)$



Marking Scheme Notes
Question 4 (c) [Scale 10C (0, 4, 7, 10)]
4: • Identifies that $y = 0$
• Some relevant substitution into the equation
• Correct answer without work shown
7: • Substantially correct work at solving for x, e.g. works incorrectly with $(x - 4)^2 = 16$
• Correct answer without the co-ordinates of P identified
Note: A correct answer with a correct geometrical explanation, award full credit