LC 2013 (Set D): Paper 1

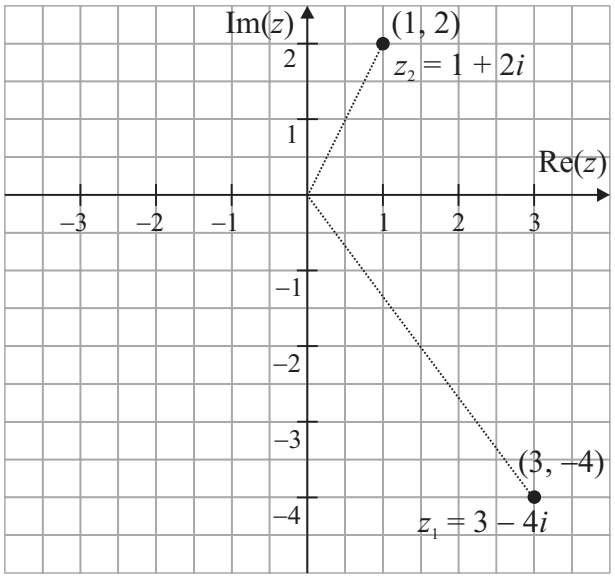
Question 1 (25 marks)

Question 1 (a)

$z_1 = 3 - 4i$

$z_2 = 1 + 2i$

z_1 and z_2 are plotted on the graph shown.



Question 1 (b)

Answer: Yes

Reason: The distance from the origin to z_1 is greater than the distance from the origin to z_2.

Marking Scheme Notes

Question 1 (a) [Scale 5C (0, 2, 4, 5)]

2: • A correct real or imaginary part plotted for either point
• Both numbers plotted correctly but real and imaginary axes interchanged

4: • Plots one point correctly

Question 1 (b) [Scale 5C (0, 2, 4, 5)]

2: • Correct answer given without a reason or reason given is without merit
• Answer not given, or incorrect, but some merit in the reason such as reference to distance

4: • Correct answer with a reason given that refers to distance
• A fully correct reason without an answer given

Question 1 (c)

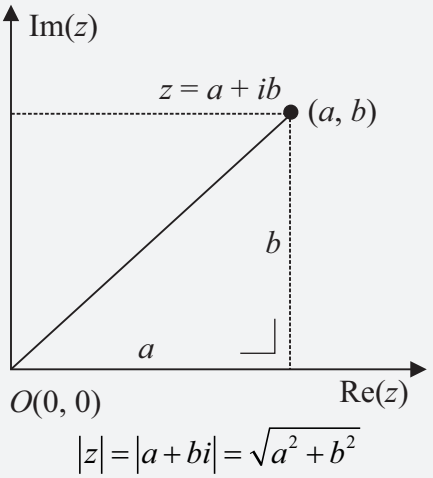
$$|z_1| = |3 - 4i|$$
$$= \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

$$|z_2| = |1 + 2i|$$
$$= \sqrt{1^2 + 2^2}$$
$$= \sqrt{1 + 4} = \sqrt{5}$$

$$5 > \sqrt{5} \Rightarrow |z_1| > |z_2|$$

Formula: Complex Numbers

Modulus $|z|$



$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Marking Scheme Notes
Question 1 (c) [Scale 5C (0, 2, 4, 5)]
2: • Correct modulus formula written.
• Each point joined to origin.
• Use of z_1 or z_2.
4: • One modulus value calculated correctly.
• Correct substitution into formula for both numbers but neither value correct.
Note: Both modulus values correct, without comparison shown, award full credit.

Question 1 (d)

$$\frac{z_1}{z_2} = \frac{3-4i}{1+2i}$$ [Multiply above and below by the conjugate of the denominator.]

$$= \frac{3-4i}{1+2i} \times \frac{1-2i}{1-2i}$$

$$= \frac{3-6i-4i+8i^2}{1-2i+2i-4i^2}$$

$$= \frac{3-10i-8}{1+4} = \frac{-5-10i}{5}$$

$$= -1-2i$$

Formula: Complex Numbers
Conjugates $\overline{z}$

$z = a+bi \Rightarrow \overline{z} = a-bi$

Multiplying a complex number by its conjugate:

$z\overline{z} = (a+bi)(a-bi) = a^2+b^2$

Marking Scheme Notes
Question 1 (b) [Scale 10C* (0, 3, 7, 10)]
3: • Correct substitution for z_1 and/or z_2
7: • Multiplication above and below by correct conjugate shown and some subsequent work
* Penalise one mark for not writing correct answer in the form $-1 - 2i$