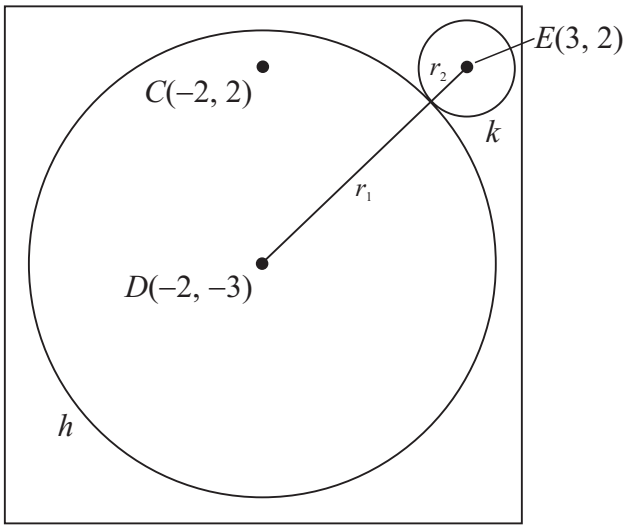
LC 2014: Paper 2

Question 9 (60 marks)



Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r

$$(x-h)^2+(y-k)^2=r^2$$

Given equation
$x^2+y^2+2gx+2fy+c=0$

Centre $(-g, -f)$

Radius $\sqrt{g^2+f^2-c}$

Question 9 (a) (i)

$h: x^2+y^2+4x+6y-19=0$

Centre $D(-2, -3)$

$$r_1=\sqrt{(-2)^2+(-3)^2-(-19)}$$
$$=\sqrt{4+9+19}$$
$$=\sqrt{32}=4\sqrt{2}$$

Question 9 (a) (ii)

$E(3, 2)$, $D(-2, -3)$, $r_1=4\sqrt{2}$

$|ED|=r_1+r_2$

$|ED|=\sqrt{(-2-3)^2+(-3-2)^2}=\sqrt{50}=5\sqrt{2}$

$\therefore 5\sqrt{2}=4\sqrt{2}+r_2$

$r_2=\sqrt{2}$

Marking Scheme Notes

Question 9 (a) (i) [Scale 15C (0, 5, 10, 15)]

5: • Effort at relating one or more coefficients of given equation to general equation of circle
• Effort at completing square(s)

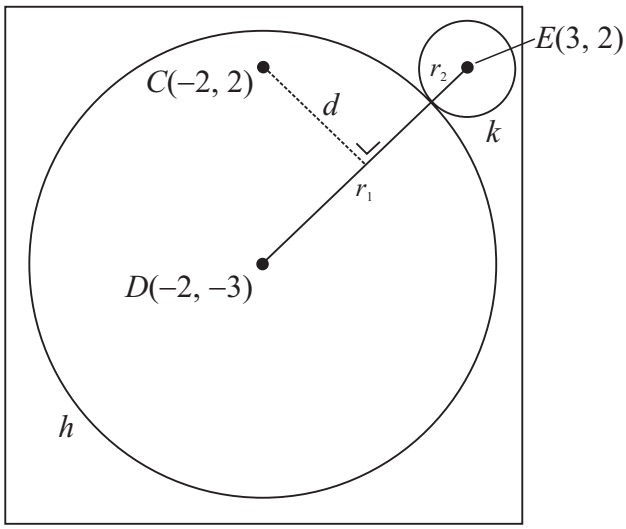
10: • Either radius or centre correct
• Substantive work with one critical error

Question 9 (a) (ii) [Scale 10C (0, 3, 7, 10)]

3: • Effort at finding $|DE|$
• Length of line segment formula
• Indicates some understanding of $r_1+r_2=|DE|$

7: • $r_1+r_2=|DE|$ or equivalent with known values substituted

Question 9 (a) (iii)



$D(-2,-3),\ E(3,2)$

$$m=\frac{2-(-3)}{3-(-2)}=\frac{5}{5}=1$$

$$y-2=1(x-3)$$

$$y-2=x-3$$

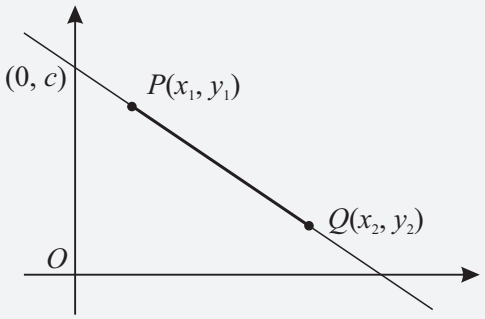
$$x-y-1=0$$

$$d=\frac{|(-2)-(2)-1|}{\sqrt{1^2+(-1)^2}}=\frac{|-5|}{\sqrt{2}}=\frac{5\sqrt{2}}{2}$$

$$|DE|=5\sqrt{2}$$

$$\therefore d=\tfrac{1}{2}|DE|$$

Formulae and Tables Book
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Length of PQ [page 18]

$$|PQ|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Midpoint of PQ [page 18]

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Equation of PQ [page 18]

$$y-y_1=m(x-x_1)$$

$$y=mx+c$$

Distance from (x_1, y_1) to the line $ax + by + c = 0$ [page 19]

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Marking Scheme Notes
Question 9 (a) (iii) [Scale 10C (0, 3, 7, 10)]

3: • Slope DE
• Equation DE and stops
• Formula for slope and/or equation of DE
• Perpendicular distance formula

7: • Values inserted into perpendicular distance formula
• No conclusion stated or implied

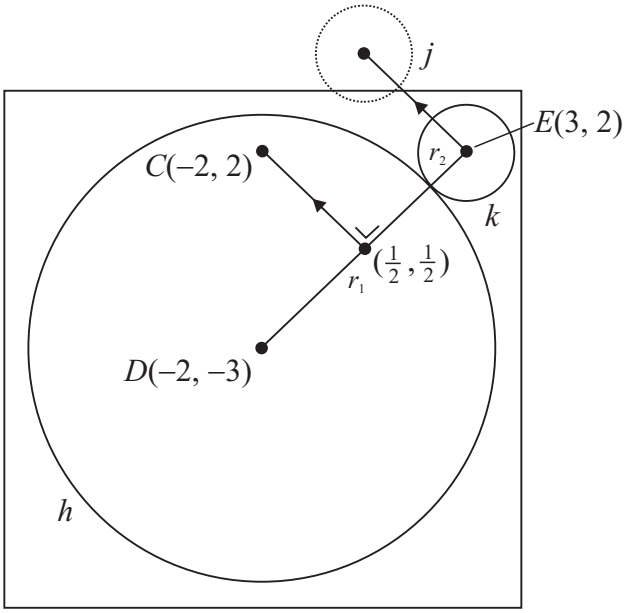
Question 9 (a) (iv)

$D(-2,-3),\ E(3,2)$

$$\text{Midpoint}=\left(\frac{-2+3}{2},\frac{-3+2}{2}\right)=\left(\frac{1}{2},-\frac{1}{2}\right)$$

$(\frac{1}{2}, -\frac{1}{2}) \to C(-2, 2)\ [-\frac{5}{2}, +\frac{5}{2}]$

$E(3, 2) \to (3-\frac{5}{2}, 2+\frac{5}{2}) = (\frac{1}{2}, \frac{9}{2})$



Equation of $j : (x-\frac{1}{2})^2 + (y-\frac{9}{2})^2 = (\sqrt{2})^2$

$(x-\frac{1}{2})^2 + (y-\frac{9}{2})^2 = 2$

Marking Scheme Notes
Question 9 (a) (iv) [Scale 5C (0, 2, 3, 5)]
2: • Effort to find midpoint of DE
• Centre found from scaled drawing
3: • Centre of j found and inserted into equation of circle i.e radius omitted

Question 9 (a) (v)

$x^2 + x^2 = (5\sqrt{2})^2$

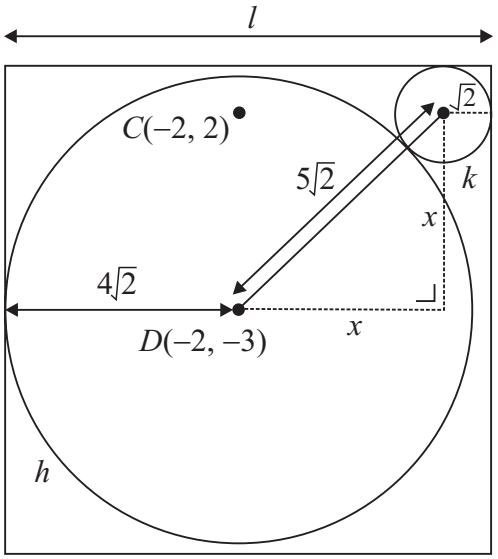
$2x^2 = 50$

$x^2 = 25$

$\therefore x = 5$

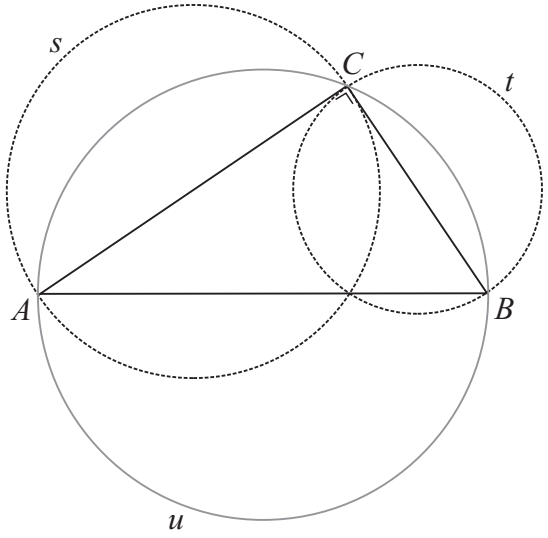
Minimum $l = 4\sqrt{2} + 5 + \sqrt{2} = 12 \cdot 07$

$\therefore l = 13$



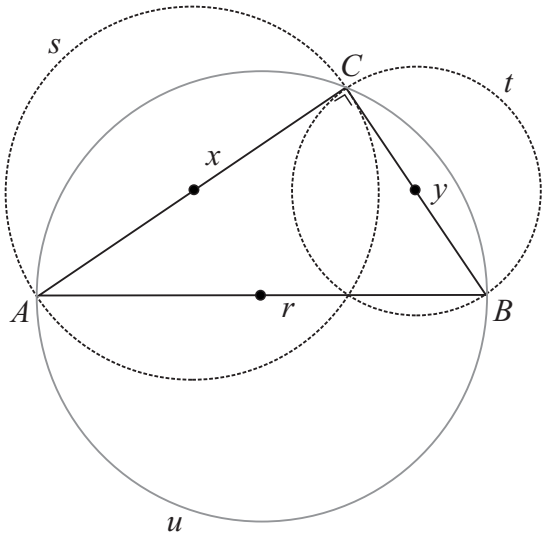
Marking Scheme Notes
Question 9 (a) (v) [Scale 5C (0, 2, 3, 5)]
2: • Effort to find F
• Indication length $r_1 + r_2 + |DE|$ (or equivalent)
3: • F found

Question 9 (b) (i)



Marking Scheme Notes
Question 9 (b) (i) [Scale 5B (0, 2, 5)]
2: • Circle containing A and B but lacking in accuracy

Question 9 (b) (ii)



Let $|AB| = r$, $|AC| = x$ and $|BC| = y$.

$x^2 + y^2 = r^2$ [Pythagoras]

Area of $s = \pi(\frac{1}{2}x)^2 = \frac{1}{4}\pi x^2$ | Area $= \pi r^2$

Area of $t = \pi(\frac{1}{2}y)^2 = \frac{1}{4}\pi y^2$

Area of $u = \pi(\frac{1}{2}r)^2 = \frac{1}{4}\pi r^2$

Area of s + Area of t

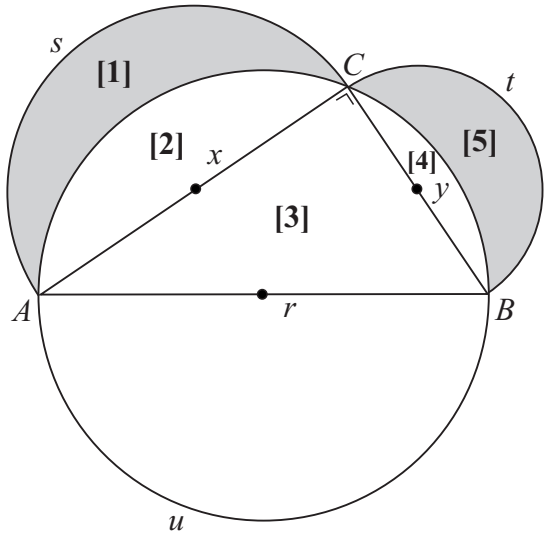
$= \frac{1}{4}\pi x^2 + \frac{1}{4}\pi y^2$

$= \frac{1}{4}\pi(x^2 + y^2)$

$= \frac{1}{4}\pi r^2 =$ Area of u

Marking Scheme Notes
Question 9 (b) (ii) [Scale 5C (0, 2, 3, 5)]
2: • Pythagoras stated or implied
• Effort at finding area of s or t or u
3: • Correct expression for area of any circle e.g $\frac{\pi}{4} \times |AB|^2$ *or* $\pi \times \frac{|AB|^2}{2}$

Question 9 (b) (iii)



Area of s + Area of t = Area of u

$\therefore \frac{1}{2}$(Area of s + Area of t) = $\frac{1}{2}$(Area of u)

Area [1] + Area [2] + Area [4] + Area [5] = Area [2] + Area [3] + Area [4]

$\therefore$ Area [1] + Area [5] = Area [3]

Marking Scheme Notes

Question 9 (b) (iii) [Scale 5C (0, 2, 3, 5)]

2: • Statement using result from (b)(ii)
• Recognising half the area of s or half the area t can be expressed in terms of two component areas
• Recognising half area of u can be expressed in terms of three components

3: • Correct expression for two of the relevant areas