

LC 2014: PAPER 1**QUESTION 9 (60 MARKS)****Question 9 (a)**

$$y = 100 - 23 = 77^\circ \text{C}$$

$$t = 0, y = 77$$

$$y = Ae^{kt}$$

$$77 = Ae^{k(0)}$$

$$77 = Ae^0$$

$$\therefore A = 77$$

Question 9 (b)

$$t = 5, y = 88 - 23 = 65$$

$$y = Ae^{kt}$$

$$65 = 77e^{5k}$$

$$e^{5k} = \frac{65}{77} \Rightarrow 5k = \ln\left(\frac{65}{77}\right)$$

$$k = \frac{1}{5} \ln\left(\frac{65}{77}\right) = -0.0339$$

MARKING SCHEME NOTES**Question 9 (a) [Scale 10C (0, 5, 7, 10)]**

- 5: • Value of y only
• Some use of 100 and/or 23
- 7: • Correct substitution into equation
• A calculated from incorrect y

Question 9 (b) [Scale 10C (0, 5, 7, 10)]

- 5: • Value of y only
• Some use of 88 and/or 23
- 7: • Correct expression for k
• k calculated from incorrect y

Question 9 (c)

$$t = ?, y = 50 - 23 = 27$$

$$y = Ae^{kt}$$

$$27 = 77e^{-0.0339t}$$

$$e^{-0.0339t} = \frac{27}{77} \Rightarrow -0.0339t = \ln\left(\frac{27}{77}\right)$$

$$\therefore t = -\frac{1}{0.0339} \ln\left(\frac{27}{77}\right) = 30.91 \approx 31 \text{ minutes}$$

MARKING SCHEME NOTES**Question 9 (c) [Scale 10C (0, 5, 7, 10)]**

- 5: • Value of y only
- 7: • Correct expression for t
• t calculated from incorrect y

Question 9 (d)

$$y = f(t) = 77e^{-0.0339t}$$

$$t = 0: f(0) = 77e^{-0.0339(0)} = 77$$

$$t = 10: f(0) = 77e^{-0.0339(10)} = 54.9 \approx 55$$

$$t = 20: f(0) = 77e^{-0.0339(20)} = 39.1 \approx 39$$

$$t = 30: f(0) = 77e^{-0.0339(30)} = 27.8 \approx 28$$

$$t = 40: f(0) = 77e^{-0.0339(40)} = 19.8 \approx 20$$

$$t = 50: f(0) = 77e^{-0.0339(50)} = 14.1 \approx 14$$

$$t = 60: f(0) = 77e^{-0.0339(60)} = 10.1 \approx 10$$

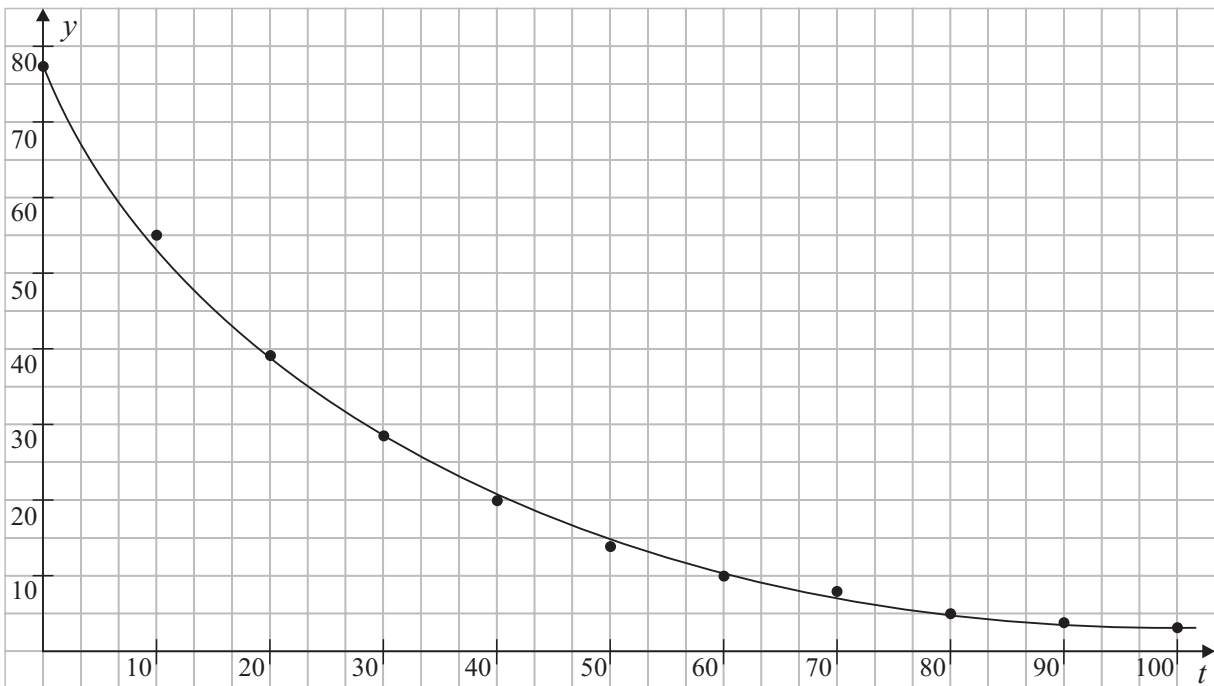
$$t = 70: f(0) = 77e^{-0.0339(70)} = 7.2 \approx 7$$

$$t = 80: f(0) = 77e^{-0.0339(80)} = 5.1 \approx 5$$

$$t = 90: f(0) = 77e^{-0.0339(90)} = 3.6 \approx 4$$

$$t = 100: f(0) = 77e^{-0.0339(100)} = 2.6 \approx 3$$

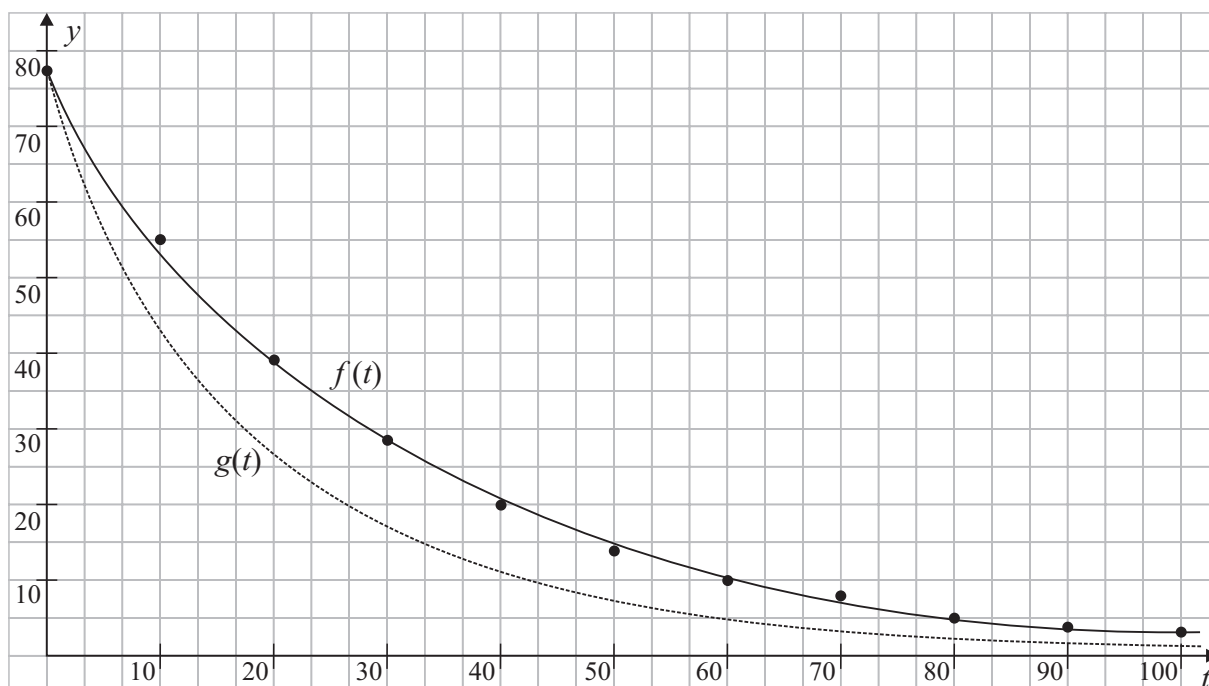
t	$f(t)$
0	77
10	55
20	39
30	28
40	20
50	14
60	10
70	7
80	5
90	4
100	3



MARKING SCHEME NOTES

Question 9 (d) [Scale 15C (0, 7, 10, 15)]

- 7: • Any one point identified
 • Graph of correct shape, even if no point correct or no point calculated
 • Accept candidates value of k
NOTE: All graphs may not be the same, due to different values of A and k
- 10: • Three points correctly plotted, but graph incomplete or no graph
NOTE: Do not accept straight line graph

Question 9 (e) (i)

Question 9 (e) (ii)

$$g(t) = 77e^{-0.05t}$$

$$m = -0.05$$

This value of m gives a larger average rate of change for the cooling time required.

MARKING SCHEME NOTES
Question 9 (e) (i) (ii) [Scale 5C (0, 3, 4, 5)]

- 3: • Any attempt at similar graph
 • No graph but correct deduction
- 4: • Correct graph plotted but graph incomplete, or no graph

Question 9 (f) (i)

$$f(t) = 77e^{-0.0339t}$$

$$f'(t) = 77(-0.0339)e^{-0.0339t} = -2.6103e^{-0.0339t}$$

$$f'(1) = -2.6103e^{-0.0339(1)} = -2.52 \text{ }^\circ\text{C per minute}$$

$$f'(10) = -2.6103e^{-0.0339(10)} = -1.86 \text{ }^\circ\text{C per minute}$$

Question 9 (f) (ii)

$$f'(t) = -2.6103e^{-0.0339t}$$

$$f''(t) = -2.6103(-0.0339)e^{-0.0339t} = \frac{0.088}{e^{0.0339t}} > 0 \text{ for all } t$$

Therefore, the rate of change of $f(t)$ is always increasing over time.

MARKING SCHEME NOTES

Question 9 (f) (i) [Scale 5C (0, 3, 4, 5)]

3: • Indication of differentiation i.e. $\frac{dy}{dt}$, $\frac{dx}{dt}$ or $f'(t)$ (i.e. differentiation with respect to t)

• Treats e as x in differentiation

4: • One value of $\frac{dy}{dt}$ indicated

Question 9 (f) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Attempt at 2nd derivative

• Attempt at deduction from numerical values

4: • Shows 2nd derivative positive
