LC 2014: Paper 1

Question 8 (50 marks)

Question 8 (a)

$y = -0\cdot013x^2 + 0\cdot624x$

$\frac{dy}{dx} = -0\cdot026x + 0\cdot624 = 0$

$0\cdot624 = 0\cdot026x$

$\therefore x = \frac{0\cdot624}{0\cdot026} = 24$

$x = 24 : y = -0\cdot013(24)^2 + 0\cdot624(24) = 7\cdot488$

Therefore, $C(24, 7\cdot488)$.

> **Find turning points (local maxiumum/minimum)**
> Put $\frac{dy}{dx} = 0$ and solve for x

> **Marking Scheme Notes**
> **Question 8 (a) [Scale 15C (0, 7, 10, 15)]**
> **7**: • Identifies $x = 24$
> • Any correct differentiation
> **10**: • Substitutes $x = 24$ in $f(x)$

Question 8 (b)

Equation of walking deck: $y = 5$
Equation of parabola: $y = -0\cdot013x^2 + 0\cdot624x$
Find the points of intersection D and E.

$-0\cdot013x^2 + 0\cdot624x = 5$

$-0\cdot013x^2 + 0\cdot624x - 5 = 0$

$a = -0\cdot013,\ b = 0\cdot624,\ c = -5$

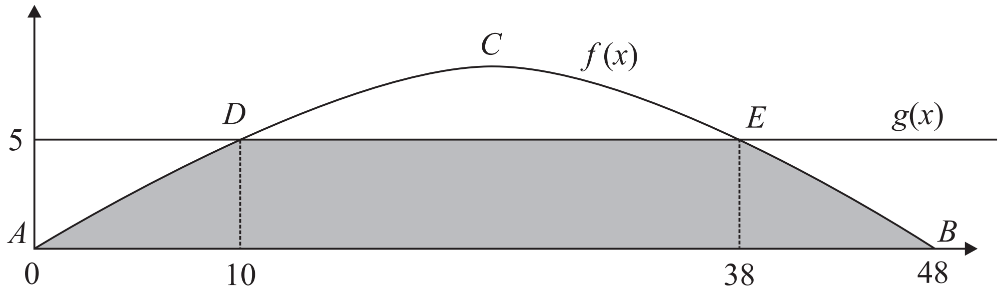
$x = \frac{-0\cdot624 \pm \sqrt{0\cdot624^2 - 4(-0\cdot013)(-5)}}{2(-0\cdot013)} \approx 10,\ 38$

$\therefore D(10, 5),\ E(38, 5)$

> **Formulae and Tables Book**
> **Algebra: Roots of the quadratic equation**
> $ax^2 + bx + c = 0$ [page 20]
> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

> **Marking Scheme Notes**
> **Question 8 (b) [Scale 10C (0, 5, 7, 10)]**
> **5**: • Recognition of line $y = 5$
> **7**: • Values in quadratic formula
> • Gets x values only
> **Notes**: Accept $x = 10$ and $x = 38$ by trial and error from correct quadratic for full marks
> NO CREDIT: Uses $x = 5$

Question 8 (c)



$$f(x) = -0 \cdot 013x^2 + 0 \cdot 624x$$
$$g(x) = 5$$

Find the area under the $f(x)$ curve between 0 and 10, multiply it by 2 (as it is the same as the area under the same curve between 38 and 48) and add on the area of the rectangle in the middle. The area of the rectangle is obtained by integrating $g(x)$ between 10 and 38.
Obviously, there is an easier way to find the area of a rectangle (length by breadth = 28 × 5 = 140), but you were asked to use integration methods. I think, either way, full marks will be given.

$$\text{Shaded area } = 2\int_0^{10} (-0 \cdot 013x^2 + 0 \cdot 624x)\, dx + \int_{10}^{38} 5\, dx$$
$$= 2\left[-\frac{0 \cdot 013x^3}{3} + \frac{0 \cdot 624x^2}{2}\right]_0^{10} + [5x]_{10}^{38}$$
$$= 2\left[-\frac{0 \cdot 013x^3}{3} + 0 \cdot 312x^2\right]_0^{10} + 5[x]_{10}^{38}$$
$$= 2\left[\left(-\frac{0 \cdot 013(10)^3}{3} + 0 \cdot 312(10)^2\right) - 0\right] + 5(38 - 10)$$
$$= 2\left[\frac{403}{15}\right] + 140 \approx 194 \text{ m}^2$$

Marking Scheme Notes
Question 8 (c) [Scale 10D (0, 3, 7, 8, 10)]
3: • Any area formula
• Area under curve from *A* to *B*
• Area under curve from *D* to *E*
• Correct limits
7: • Any correct integration
• One correct area only
• Area under curve from *A* to *B* minus area under curve from *D* to *E*
8: • Limits substituted but not evaluated (must state that area of rectangle is 140)
• Both areas
OR
3: • Any one area
7: • Translation
8: • 3rd area

Question 8 (d)

$y = -0\cdot013x^2 + 0\cdot624x$

$y = -0\cdot013(x^2 - 48x)$

$y = -0\cdot013(x^2 - 48x + (-24)^2 - 576)$ [Completing the square]

$y = -0\cdot013(x^2 - 48x + (-24)^2 - 576)$

$y = -0\cdot013((x-24)^2 - 576)$

$y = -0\cdot013(x-24)^2 + 7\cdot488$

$y - 7\cdot488 = -0\cdot013(x-24)^2$

Question 8 (e)

$y = -0\cdot013x^2 + 0\cdot624x$

Local maximum $= (24,\ 7\cdot488)$

$y - 7\cdot488 = -0\cdot013(x-24)^2$

$y = -2x^2 + \ldots\ldots$

Local maximum $= (3,\ -4)$

$\therefore\ y - (-4) = -2(x-3)^2$

$y + 4 = -2(x-3)^2$

$y + 4 = -2(x^2 - 6x + 9)$

$y + 4 = -2x^2 + 12x - 18$

$y = -2x^2 + 12x - 22$

Marking Scheme Notes

Question 8 (d) [Scale 10C (0, 5, 7, 10)]

5: • Common factor of –0·013 identified
• Attempt at equating like to like
• Attempt at completing square

7: • Values of two of the constants found
• *p* correct and completion of square correct

Question 8 (e) [Scale 5B (0, 3, 5)]

3: One correct value for the equivalent of *k*, *p* or *h* in equation