

LC 2014: PAPER 1**QUESTION 6 (25 MARKS)****Question 6 (a) (i)**

$$T_n = \ln a^n = n \ln a$$

$$T_1 = \ln a, T_2 = 2 \ln a, T_3 = 3 \ln a$$

$$T_3 - T_2 = 3 \ln a - 2 \ln a = \ln a$$

$$T_2 - T_1 = 2 \ln a - \ln a = \ln a$$

$$\therefore T_3 - T_2 = T_2 - T_1$$

Therefore, T_1, T_2 and T_3 are in arithmetic sequence.

Question 6 (a) (ii)

$$T_n = \ln a^n = n \ln a$$

$$T_{n+1} = (n+1) \ln a$$

$$T_{n+1} - T_n = (n+1) \ln a - n \ln a$$

$$= n \ln a + \ln a - n \ln a$$

$$= \ln a = \text{Constant } (d)$$

Therefore, the sequence is arithmetic with common difference $d = \ln a$.

FORMULAE AND TABLES BOOK**Indices and logs [page 21]**

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a(x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a \left(\frac{1}{x} \right) = -\log_a x$$

MARKING SCHEME NOTES

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

Question 6 (a) (i) [Scale 10C (0, 5, 7, 10)]

5: • Only one term correct

7: • Either $(T_2 - T_1)$ or $(T_3 - T_2)$ correct

Question 6 (a) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Uses two consecutive general terms

• Recognition of common difference and no more

4: • Shows series arithmetic but does not state common difference

Question 6 (b)

$$T_1 + T_2 + T_3 + \dots + T_{98} + T_{99} + T_{100} = S_{100} = 10\,100$$

$$T_1 = \ln a, d = \ln a, n = 100$$

$$S_{100} = \frac{100}{2} [2 \ln a + 99 \ln a] = 10\,100$$

$$50[101 \ln a] = 10\,100$$

$$\ln a = 2$$

$$\therefore a = e^2$$

FORMULAE AND TABLES BOOK
Sequences and series: Arithmetic series

[page 22]

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

a is the first term

d is the common difference

MARKING SCHEME NOTES

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

Question 6 (b) [Scale 5C (0, 3, 4, 5)]

- 3: • Writes three or more terms in form of n and $\ln a$
 • Correct AP formula stated
 • Correct T_n formula
- 4: • Correct substitution into formula
 • $\ln a = 2$ and does not finish

NOTE: accept $a = e^2$ for full marks

Question 6 (c)

| LHS | | RHS |
|---|--|--|
| $T_1 + T_2 + T_3 + \dots + T_{10} + 100d$ | | $T_{11} + T_{12} + T_{13} + \dots + T_{20}$ |
| $= S_{10} + 100d$ | | $= S_{20} - S_{10}$ |
| $= \frac{10}{2}[2 \ln a + 9 \ln a] + 100 \ln a$ | | $= \frac{20}{2}[2 \ln a + 19 \ln a] - \frac{10}{2}[2 \ln a + 9 \ln a]$ |
| $= 5[11 \ln a] + 100 \ln a$ | | $= 10[21 \ln a] - 5[11 \ln a]$ |
| $= 155 \ln a$ | | $= 210 \ln a - 55 \ln a$ |
| | | $= 155 \ln a$ |

MARKING SCHEME NOTES

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

Question 6 (c) [Scale 5C (0, 3, 4, 5)]

- 3: • Recognising $T_{11} = T_1 + 10d$ or similar work
- 4: • LHS correct in terms of $\ln a$
 • RHS correct in terms of $\ln a$

NOTE: log is not needed in first solution box