LC 2014: Paper 1

Question 5 (25 marks)

Question 5 (a)

$$\int 5\cos 3x\,dx$$
$$= 5\int \cos 3x\,dx$$
$$= \tfrac{5}{3}\sin 3x + c$$

Formulae and Tables Book
Calculus: Integrals [page 26]

$$\int \cos x\,dx = \sin x + c$$
$$\int \cos(ax+b)\,dx = \tfrac{1}{a}\sin(ax+b) + c$$

Marking Scheme Notes
Question 5 (a) [Scale 5B (0, 3, 5)]
3: • Some correct integration
• Integrand does not contain c
• c only

Question 5 (b) (i)

$$\frac{dy}{dx} = 2x - 2$$
$$\int dy = \int (2x-2)\,dx$$
$$y = \frac{2x^2}{2} - 2x + c$$
$$= x^2 - 2x + c$$

Formulae and Tables Book
Calculus: Integrals [page 26]

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$$

(–2, 0) is on the curve.

$$y = x^2 - 2x + c$$
$$x = -2,\ y = 0 \Rightarrow 0 = (-2)^2 - 2(-2) + c$$
$$0 = 4 + 4 + c$$
$$\therefore c = -8$$
$$\therefore y = f(x) = x^2 - 2x - 8$$

Marking Scheme Notes
Question 5 (b) (i) [Scale 10C (0, 5, 7, 10)]
5: • Some correct integration
• Integrand does not contain c
• c only
• $\frac{dy}{dx} = 2x - 2$ or $\frac{dy}{dx}$ = slope of tangent

7: • Substitutes (–2, 0) but c not simplified
Note: must have 'c' in equation to get high partial marks

Question 5 (b) (ii)

$$\bar{f} = \frac{1}{3-0}\int_0^3 (x^2 - 2x - 8)\,dx$$

$$= \frac{1}{3}\left[\frac{x^3}{3} - \frac{2x^2}{2} - 8x\right]_0^3 = \frac{1}{3}\left[\frac{x^3}{3} - x^2 - 8x\right]_0^3$$

$$= \frac{1}{3}\left[\left(\frac{3^3}{3} - 3^2 - 8(3)\right) - 0\right]$$

$$= \tfrac{1}{3}[9 - 9 - 24] = -8$$

Formula
Average value of f

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Formulae and Tables Book
Calculus: Integrals [page 26]

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$$

Marking Scheme Notes
Question 5 (b) (ii) [Scale 10C (0, 5, 7, 10)]

5: • Correct formula only
• Some correct integration
• Indication of integration with correct limits
• If only values used e.g. $f(0), f(1), f(2)$ etc. when $0 \leq x \leq 3$, give Low Partial Credit for two or more values

7: • Limits inserted into function but not calculated
• $\frac{1}{(b-a)}$ missing from formula

Note: NO CREDIT – differentiation
NO CREDIT – no integration