LC 2014: Paper 1

Question 3 (25 marks)

Question 3 (a)

> **Steps to Proof by Induction**
> **1**. Prove result is true for some starting value of $n \in \mathbb{N}$.
> **2**. Assume result is true for $n = k$.
> **3**. Prove result is true for $n = (k+1)$, using step 2.

Required to Prove: $1+2+3+............+n = \dfrac{n(n+1)}{2}$

Step 1: Prove the result is true for $n = 1$.

$$1 = \frac{1(1+1)}{2}$$

$$1 = \frac{1(2)}{2}$$

$1 = 1$ [Therefore, true for $n = 1$.]

Step 2: Assume it is true for $n = k$.

$$\underline{1+2+3+............+k} = \frac{k(k+1)}{2}$$

Step 3: Prove it is true for $n = k + 1$.

Prove $\underline{(1+2+3+............+k)} + (k+1) = \dfrac{(k+1)(k+2)}{2}$

Use the result in Step 2 to prove Step 3.

$$(1+2+3+............+k)+(k+1)$$

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= (k+1)\left[\frac{k}{2}+1\right]$$

$$= (k+1)\left[\frac{k+2}{2}\right]$$

$$= \frac{(k+1)(k+2)}{2}$$

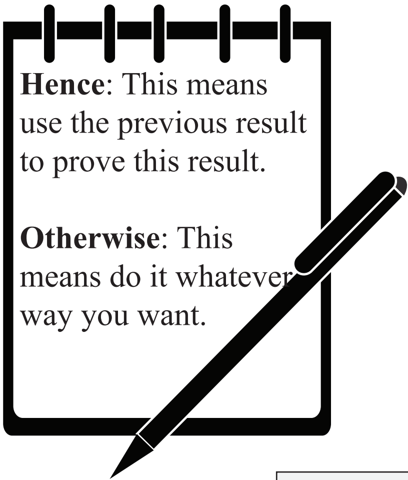
Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n \in \mathbb{N}$.

MARKING SCHEME NOTES
Question 3 (a) [Scale 10D (0, 3, 7, 8, 10)]
3: • One correct step in induction
• Statement $P(1)$ true
7: • Uses $(k+1)$ term on LHS
• Some work with $(k+1)$ term
8: • Correct RHS
• No conclusion

Question 3 (b)

Hence:

$$2+4+6+\ldots+2n$$
$$=2(1+2+3+\ldots+n)$$
$$=2\left[\frac{n(n+1)}{2}\right]$$
$$=n^2+n$$



Otherwise:

$$S_n=2+4+6+\ldots+2n\ [a=2,\ d=2,\ n \text{ terms}]$$
$$S_n=\tfrac{n}{2}[2(2)+(n-1)(2)]$$
$$=\tfrac{n}{2}[4+2n-2]$$
$$=\tfrac{n}{2}[2+2n]$$
$$=n[n+1]$$
$$=n^2+n$$

FORMULAE AND TABLES BOOK
Sequences and series: Arithmetic series
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$$S_n=\frac{n}{2}[2a+(n-1)d]$$

a is the first term
d is the common difference

MARKING SCHEME NOTES
Question 3 (b) [Scale 10D (0, 3, 7, 8, 10)]
3: • Recognising 2 as common factor
• $\frac{n}{2}(n+1)$
7: • Correct equation from (a)
• Taking 2 out of series
8: • Work not completed

OR

Question 3 (b) [Scale 10D (0, 3, 7, 8, 10)]
3: • Recognition of $a=2$
• Recognition of $d=2$
• Correct AP formula only
7: • Some substitution into correct formula
8: • Work not fully simplified
• Answer not in required form
• S_n missing

Question 3 (c)

Sum of the first n **odd** natural numbers S + Sum of the first n **even** natural numbers
= Sum of the first $2n$ natural numbers

$$S+(n^2+n)=\frac{2n(2n+1)}{2}$$

$$S=n(2n+1)-(n^2+n)$$

$$=2n^2+n-n^2-n$$

$$=n^2$$

Marking Scheme Notes

Question 3 (c) [Scale 5B (0, 3, 5)]

3:
- $S_B - S_A$ indicated
- $S_B + S_A$
- Use of correct series from (b)

Note: Must use result from (a) and (b) here