LC 2015: Paper 1

Question 9 (50 marks)

Question 9 (a)

$f(t)=12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}t\right)$ ← Replace t by 76 in this function and use your calculator in radian mode.

$f(76)=12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}\times 76\right)=16\cdot 835 \text{ hours}=16 \text{ h } 50 \text{ mins}$

> **Marking Scheme Notes**
> **Question 9 (a) [Scale 10C (0, 4, 8, 10)]**
> **4**: • Uses $t = 76$
> **8**: • Correct substitution
> **Note**: Using $\pi = 90^{o}$ ⇒ one error, but do not penalise again in (b)

Question 9 (b)

$f(t)=12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}t\right)=15$ ← Put $f(t)$ equal to 15 and use your calculator to find t.

$4\cdot 75\sin\left(\frac{2\pi}{365}t\right)=2\cdot 75$

$\sin\left(\frac{2\pi}{365}t\right)=\frac{2\cdot 75}{4\cdot 75}=\frac{11}{19}$

$t=\frac{365}{2\pi}\sin^{-1}\left(\frac{11}{19}\right)\approx 35\cdot 87 \text{ days}$

36 days after 21st March is 26th April

> **Marking Scheme Notes**
> **Question 9 (b) [Scale 10C (0, 4, 8, 10)]**
> **4**: • Correct $f(t)$
> • $f(15)$ substituted
> **8**: • Correct equation with t only
> **Note**: Accept 35 or 36 substituted correctly and tested

Question 9 (c)

$f(t)=12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}t\right)$

$f'(t)=4\cdot 75\cos\left(\frac{2\pi}{365}t\right)\times\frac{2\pi}{365}=\frac{19\pi}{730}\cos\left(\frac{2\pi}{365}t\right)$

> **Formulae and Tables Book**
> **Calculus: Integrals** [page 25]
> $y=\sin x\Rightarrow\frac{dy}{dx}=\cos x$
> $y=\sin f(x)\Rightarrow\frac{dy}{dx}=\cos f(x)\times f'(x)$

> **Marking Scheme Notes**
> **Question 9 (c) [Scale 10B (0, 5, 10)]**
> **5**: • Any correct differentiation (note: '0' could be correct differentiation here)
> **Note**: Substituting 180° for π ⇒ one error

Question 9 (d)

> **Find turning points (local maxiumum/minimum)**
> Put $\frac{dy}{dx}=0$ and solve for x

$$f'(t)=0 \Rightarrow \frac{19\pi}{730}\cos\left(\frac{2\pi}{365}t\right)=0$$

$$\cos\left(\frac{2\pi}{365}t\right)=0$$

$$\frac{2\pi}{365}t=\cos^{-1}0=\frac{\pi}{2}$$

$$t=\frac{365}{4}$$

$$f(t)=12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}t\right)$$

$$\therefore f\left(\frac{365}{4}\right)=12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}\times\frac{365}{4}\right)=12\cdot 25+4\cdot 75\sin\left(\frac{\pi}{2}\right)=12\cdot 25+4\cdot 75=17 \text{ hours}$$

Marking Scheme Notes
Question 9 (d) [Scale 10D (0, 2, 5, 8, 10)] – both solutions
2: • $f'(t)=0$
5: • Value of t
8: • Value of t substituted into $f(t)$
• $f(t)$ maximum when $\sin\theta=1$
Note: Accept 91 or 92 substituted and evaluated correctly for full marks

Question 9 (e)

> **Formulae and Tables Book**
> **Calculus: Integrals** [page 26]
> $\int \sin x\,dx=-\cos x+c$
> $\int \sin(ax+b)\,dx=-\frac{1}{a}\cos(ax+b)+c$

$$\bar{L}=\frac{1}{184-0}\int_0^{184}\left[12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}t\right)\right]dt$$

$$=\frac{1}{184}\left[12\cdot 25t-4\cdot 75\cos\left(\frac{2\pi}{365}t\right)\times\frac{365}{2\pi}\right]_0^{184}$$

$$=\frac{1}{184}\left[12\cdot 25t-\frac{4\cdot 75\times 365}{2\pi}\cos\left(\frac{2\pi}{365}t\right)\right]_0^{184}$$

$$=\frac{1}{184}\left[\left(12\cdot 25(184)-\frac{4\cdot 75\times 365}{2\pi}\cos\left(\frac{2\pi}{365}\times 184\right)\right)-\left(12\cdot 25(0)-\frac{4\cdot 75\times 365}{2\pi}\cos\left(\frac{2\pi}{365}(0)\right)\right)\right]$$

$$=\frac{1}{184}\left[\left(12\cdot 25(184)-\frac{4\cdot 75\times 365}{2\pi}\cos\left(\frac{2\pi}{365}\times 184\right)+\frac{4\cdot 75\times 365}{2\pi}\right)\right]$$

$$=15\cdot 2488 \text{ hours}=15 \text{ hours } 15 \text{ minutes}$$

Marking Scheme Notes
Question 9 (e) [Scale 10D (0, 2, 5, 8, 10)]
2: • Correct expression in x or t
• Correct formula
• Correct limits
5: • Any correct integration
8: • Correct integration and effort at substitution
Note: Integration with one error but finished correctly gets High Partial Credit