LC 2015: Paper 2

Question 8 (65 marks)

P(Successful on first free throw) = 0·7
P(Unsuccessful on first free throw) = 0·3
P(Successful if he was successful on previous free throw) = 0·8
P(Unsuccessful if he was successful on previous free throw) = 0·2
P(Successful if he was unsuccessful on previous free throw) = 0·6
P(Unsuccessful if he was unsuccessful on previous free throw) = 0·4

Question 8 (a)

$P(\text{S, S, S}) = 0{\cdot}7 \times 0{\cdot}8 \times 0{\cdot}8 = 0{\cdot}448 = \frac{56}{125}$

Marking Scheme Notes
Question 8 (a) [Scale 10C (0, 4, 8, 10)]
4: • One correct probability
8: • Identifies all three probabilities correctly
• Three probabilities multiplied of which two are correct

Question 8 (b)

$P(\text{U, U, S}) = 0{\cdot}3 \times 0{\cdot}4 \times 0{\cdot}6 = 0{\cdot}072 = \frac{9}{125}$

Marking Scheme Notes
Question 8 (b) [Scale 10C (0, 4, 8, 10)]
4: • One correct probability
8: • Identifies all three probabilities correctly
• Three probabilities multiplied of which two are correct

Question 8 (c)

P(Successful on third free throw)
= P(S, S, S) **or** P(U, U, S) **or** P(S, U, S) **or** P(U, S, S)
= P(S, S, S) + P(U, U, S) + P(S, U, S) + P(U, S, S)

$$= \frac{56}{225} + \frac{9}{125} + 0 \cdot 7 \times 0 \cdot 2 \times 0 \cdot 6 + 0 \cdot 3 \times 0 \cdot 6 \times 0 \cdot 8$$

$$= \frac{56}{225} + \frac{9}{125} + \frac{21}{250} + \frac{18}{125}$$

$$= \frac{56}{125} + \frac{9}{125} + \frac{21}{250} + \frac{18}{125} = \frac{187}{250} = 0 \cdot 748$$

Marking Scheme Notes
Question 8 (c) [Scale 15D (0, 4, 7, 11, 15)]
4: • Lists one new way
7: • Full listing only
• One new probability
11: • Sum of three probabilities
• Identifies all four probabilities correctly

Question 8 (d) (i)

p_{n+1} = Probability that Michael is successful on the throw after his *n*th throw

p_n = Probability that Michael is successful on his *n*th throw

Probability that Michael is successful on the throw after his *n*th throw
= Probability that Michael is successful on his previous throw **or** Probability that Michael is unsuccessful on his previous throw
$\therefore p_{n+1} = 0{\cdot}8 \times p_n + 0{\cdot}6(1 - p_n) = 0{\cdot}8p_n + 0{\cdot}6 - 0{\cdot}6p_n = 0{\cdot}6 + 0{\cdot}2p_n$

Marking Scheme Notes
Question 8 (d) (i) [Scale 5C (0, 2, 4, 5)]
2: • Indicates P(S, S) and/or P(U, S) or equivalent
4: • Substitution into equation for p_{n+1}

Question 8 (d) (ii)

$p_{n+1} \approx p_n \approx p$
$\therefore p_{n+1} = 0{\cdot}6 + 0{\cdot}2p_n = p_n$
$0{\cdot}6 = 0{\cdot}8p_n$
$\therefore p_n = 0{\cdot}75$

Marking Scheme Notes
Question 8 (d) (ii) [Scale 5C (0, 2, 4, 5)]
2: • Indicates P(S, S) and/or P(U, S) or equivalent
4: • Substitution into equation for p_{n+1}

Question 8 (e) (i)

$$a_n = p - p_n = 0\cdot 75 - p_n$$

$$\frac{a_{n+1}}{a_n} = \frac{0\cdot 75 - p_{n+1}}{0\cdot 75 - p_n} = \frac{0\cdot 75 - 0\cdot 6 - 0\cdot 2p_n}{0\cdot 75 - p_n}$$

$$= \frac{0\cdot 15 - 0\cdot 2p_n}{0\cdot 75 - p_n}$$

$$= \frac{15 - 20p_n}{75 - 100p_n}$$

$$= \frac{3 - 4p_n}{15 - 20p_n}$$

$$= \frac{3 - 4p_n}{5(3 - 4p_n)} = \frac{1}{5}$$

Therefore, a_n is a geometric sequence.

Marking Scheme Notes
Question 8 (e) (i) [Scale 5C (0, 2, 4, 5)]
2: • a_{n+1} in terms of p and p_{n+1}
• $\frac{a_{n+1}}{a_n}$ in terms of p, p_n and p_{n+1}

4: • $\frac{a_{n+1}}{a_n}$ substituted

Question 8 (e) (ii)

$$\frac{a_{n+1}}{a_n} = \frac{1}{5} \Rightarrow a_{n+1} = \frac{1}{5} \times a_n$$

$$a_n = p - p_n = 0 \cdot 00001$$

$$n = 1 : a_1 = p - p_1 = 0 \cdot 75 - 0 \cdot 7 = 0 \cdot 05 \leftarrow$$ a_n is a geometric sequence with first term $a = 0{\cdot}05$ and common ratio $r = \frac{1}{5}$

$$\therefore a_n = a(r)^{n-1} = (0 \cdot 05)\left(\frac{1}{5}\right)^{n-1} < 0 \cdot 00001$$

$$\left(\frac{1}{5}\right)^{n-1} < \frac{1}{5000}$$

$$(n-1)\log\left(\frac{1}{5}\right) < \log\left(\frac{1}{5000}\right)$$

$$n - 1 > \frac{\log\left(\frac{1}{5000}\right)}{\log\left(\frac{1}{5}\right)} = 5 \cdot 29$$ $\leftarrow$ Reverse the inequality as log $(0{\cdot}2)$ is negative.

$$n > 6 \cdot 29$$

Smallest value of n is 7.

> **Marking Scheme Notes**
> **Question 8 (e) (ii) [Scale 5C (0, 2, 4, 5)]**
> **2**: • a_1 in numerical form
> **4**: • ar^{n-1} substituted
> • a_7 evaluated without checking a_6

Question 8 (f) (i)

$$p \approx 0 \cdot 75$$

Question 8 (f) (ii)

It would not be appropriate to consider Michaels's subsequent free throws in the game as sequence of Bernoulli trials as the probability changes from throw to throw. The events are not independent.

> **Marking Scheme Notes**
> **Question 8 (f) (i) (ii) [Scale 5B (0, 2, 5)]**
> **2**: • (i) correct only or (ii) correct only