

LC 2015 (SET B): PAPER 2**QUESTION 8 (65 MARKS)**

$$P(\text{Successful on first free throw}) = 0.7$$

$$P(\text{Unsuccessful on first free throw}) = 0.3$$

$$P(\text{Successful if he was successful on previous free throw}) = 0.8$$

$$P(\text{Unsuccessful if he was successful on previous free throw}) = 0.2$$

$$P(\text{Successful if he was unsuccessful on previous free throw}) = 0.6$$

$$P(\text{Unsuccessful if he was unsuccessful on previous free throw}) = 0.4$$

Question 8 (a)

$$P(S, S, S) = 0.7 \times 0.8 \times 0.8 = 0.448 = \frac{56}{125}$$

MARKING SCHEME NOTES**Question 8 (a) [Scale 10C (0, 4, 8, 10)]**

- 4: • One correct probability
- 8: • Identifies all three probabilities correctly
• Three probabilities multiplied of which two are correct

Question 8 (b)

$$P(U, U, S) = 0.3 \times 0.4 \times 0.6 = 0.072 = \frac{9}{125}$$

MARKING SCHEME NOTES**Question 8 (b) [Scale 10C (0, 4, 8, 10)]**

- 4: • One correct probability
- 8: • Identifies all three probabilities correctly
• Three probabilities multiplied of which two are correct

Question 8 (c)

$$P(\text{Successful on third free throw})$$

$$= P(S, S, S) \text{ or } P(U, U, S) \text{ or } P(S, U, S) \text{ or } P(U, S, S)$$

$$= P(S, S, S) + P(U, U, S) + P(S, U, S) + P(U, S, S)$$

$$= \frac{56}{225} + \frac{9}{125} + 0.7 \times 0.2 \times 0.6 + 0.3 \times 0.6 \times 0.8$$

$$= \frac{56}{225} + \frac{9}{125} + \frac{21}{250} + \frac{18}{125}$$

$$= \frac{56}{125} + \frac{9}{125} + \frac{21}{250} + \frac{18}{125} = \frac{187}{250} = 0.748$$

MARKING SCHEME NOTES**Question 8 (c) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Lists one new way
- 7: • Full listing only
• One new probability
- 11: • Sum of three probabilities
• Identifies all four probabilities correctly

Question 8 (d) (i)

p_{n+1} = Probability that Michael is successful on the throw after his n th throw

p_n = Probability that Michael is successful on his n th throw

Probability that Michael is successful on the throw after his n th throw

= Probability that Michael is successful on his previous throw **or** Probability that Michael is unsuccessful on his previous throw

$$\therefore p_{n+1} = 0.8 \times p_n + 0.6(1 - p_n) = 0.8p_n + 0.6 - 0.6p_n = 0.6 + 0.2p_n$$

MARKING SCHEME NOTES**Question 8 (d) (i) [Scale 5C (0, 2, 4, 5)]**

2: • Indicates $P(S, S)$ and/or $P(U, S)$ or equivalent

4: • Substitution into equation for p_{n+1}

Question 8 (d) (ii)

$$p_{n+1} \approx p_n \approx p$$

$$\therefore p_{n+1} = 0.6 + 0.2p_n = p_n$$

$$0.6 = 0.8p_n$$

$$\therefore p_n = 0.75$$

MARKING SCHEME NOTES**Question 8 (d) (ii) [Scale 5C (0, 2, 4, 5)]**

2: • Indicates $P(S, S)$ and/or $P(U, S)$ or equivalent

4: • Substitution into equation for p_{n+1}

Question 8 (e) (i)

$$a_n = p - p_n = 0.75 - p_n$$

$$\frac{a_{n+1}}{a_n} = \frac{0.75 - p_{n+1}}{0.75 - p_n} = \frac{0.75 - 0.6 - 0.2p_n}{0.75 - p_n}$$

$$= \frac{0.15 - 0.2p_n}{0.75 - p_n}$$

$$= \frac{15 - 20p_n}{75 - 100p_n}$$

$$= \frac{3 - 4p_n}{15 - 20p_n}$$

$$= \frac{3 - 4p_n}{5(3 - 4p_n)} = \frac{1}{5}$$

Therefore, a_n is a geometric sequence.

MARKING SCHEME NOTES**Question 8 (e) (i) [Scale 5C (0, 2, 4, 5)]**

2: • a_{n+1} in terms of p and p_{n+1}

• $\frac{a_{n+1}}{a_n}$ in terms of p , p_n and p_{n+1}

4: • $\frac{a_{n+1}}{a_n}$ substituted

Question 8 (e) (ii)

$$\frac{a_{n+1}}{a_n} = \frac{1}{5} \Rightarrow a_{n+1} = \frac{1}{5} \times a_n$$

$$a_n = p - p_n = 0.00001$$

$n = 1: a_1 = p - p_1 = 0.75 - 0.7 = 0.05 \leftarrow a_n$ is a geometric sequence with first term $a = 0.05$ and common ratio $r = \frac{1}{5}$

$$\therefore a_n = a(r)^{n-1} = (0.05)\left(\frac{1}{5}\right)^{n-1} < 0.00001$$

$$\left(\frac{1}{5}\right)^{n-1} < \frac{1}{5000}$$

$$(n-1)\log\left(\frac{1}{5}\right) < \log\left(\frac{1}{5000}\right)$$

$$n-1 > \frac{\log\left(\frac{1}{5000}\right)}{\log\left(\frac{1}{5}\right)} = 5.29 \leftarrow \text{Reverse the inequality as } \log(0.2) \text{ is negative.}$$

$$n > 6.29$$

Smallest value of n is 7.

MARKING SCHEME NOTES

Question 8 (e) (ii) [Scale 5C (0, 2, 4, 5)]

2: • a_1 in numerical form

4: • ar^{n-1} substituted

• a_7 evaluated without checking a_6

Question 8 (f) (i)

$$p \approx 0.75$$

Question 8 (f) (ii)

It would not be appropriate to consider Michaels's subsequent free throws in the game as sequence of Bernoulli trials as the probability changes from throw to throw. The events are not independent.

MARKING SCHEME NOTES

Question 8 (f) (i) (ii) [Scale 5B (0, 2, 5)]

2: • (i) correct only or (ii) correct only