LC 2015: Paper 1

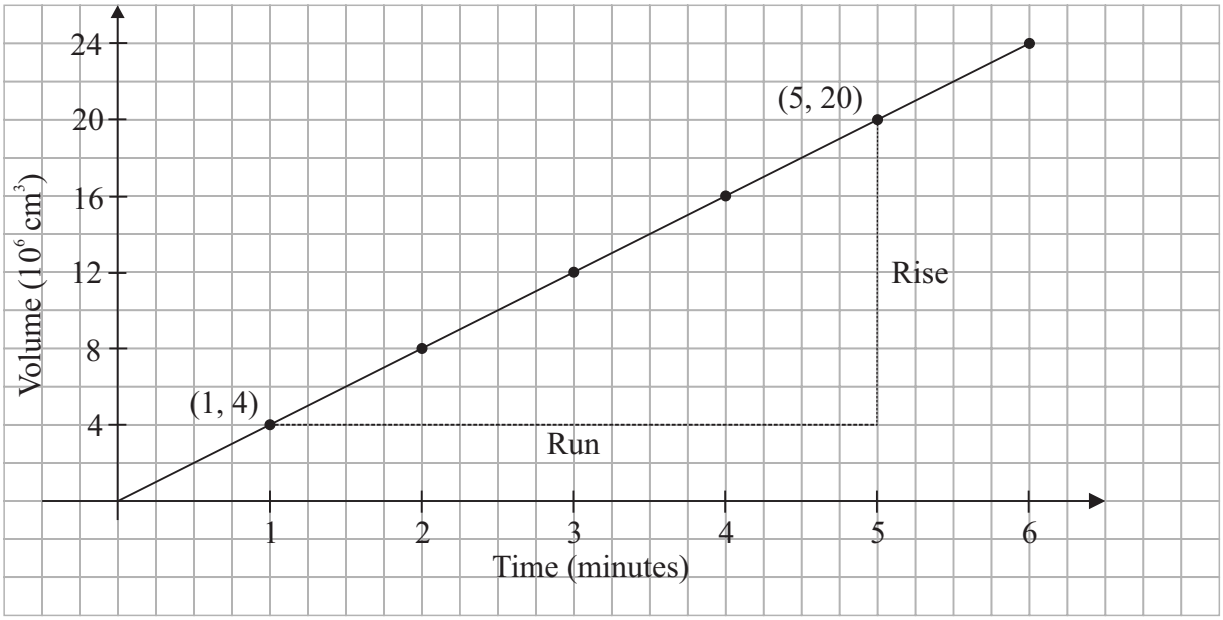
Question 8 (50 marks)

Question 8 (a) (i)

Volume $V = 4 \times 10^6$ cm^3 per minute
Time $t = 1$ minute $\Rightarrow V = 4 \times 10^6$ cm^3
Time $t = 2$ minutes $\Rightarrow V = 4 \times 10^6 \times 2 = 8 \times 10^6$ cm^3

Time (minutes)	1	2	3	4	5	6
Volume (10^6 cm^3)	4	8	12	16	20	24

Question 8 (a) (ii)



Question 8 (a) (iii)

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{20\times10^6 - 4\times10^6}{5-1} = \frac{16\times10^6}{4} = 4\times10^6$$

$V = 4\times10^6 t$ [$y = mx$]

Marking Scheme Notes

Question 8 (a) (i) [Scale 5B (0, 2, 5)]

2: • One correct box

Question 8 (a) (ii) [Scale 5B (0, 2, 5)]

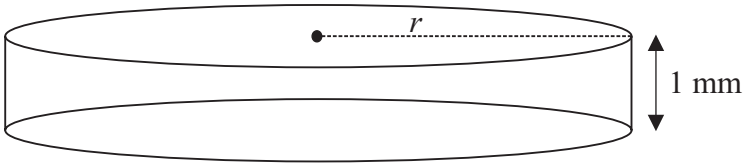
0: • Bar chart

2: • At least two points plotted

Question 8 (a) (iii) [Scale 5B (0, 2, 5)]

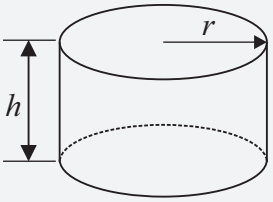
2: • Incomplete equation for volume
• V = any function of t
• Attempt at finding slope

Question 8 (b) (i)



$V = \pi r^2 h = \pi r^2 (0 \cdot 1) = \frac{1}{10}\pi r^2 \text{ cm}^3$

Formulae and Tables Book
Surface area and volume:
Cylinder [page 10]



$A = 2\pi rh$

$V = \pi r^2 h$

Question 8 (b) (ii)

$V = 4 \times 10^6 t \Rightarrow \frac{dV}{dt} = 4 \times 10^6$

$V = \frac{1}{10}\pi r^2 \Rightarrow \frac{dV}{dt} = \frac{1}{5}\pi r \times \frac{dr}{dt} = 4 \times 10^6$

$r = 50 \text{ m} = 5000 \text{ cm} : 4 \times 10^6 = \frac{1}{5}\pi(5000) \times \frac{dr}{dt}$

$$4000 = \pi \times \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{4000}{\pi} \text{ cm/minute} = 1273 \cdot 3 \text{ cm/minute}$$

Marking Scheme Notes
Question 8 (b) (i) [Scale 5B (0, 2, 5)]
2: • Correct volume formula
• Converting mm to cm

Question 8 (b) (ii) [Scale 10D (0, 2, 5, 8, 10)]
2: • Mentions a relevant rate of change
5: • Gets $\frac{dr}{dt}$ from $\frac{dV}{dr}$ and $\frac{dV}{dt}$
• Writing down chain rule
8: • Substitution of values

Question 8 (c)

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \times \frac{dr}{dt}$

$\frac{1}{5}\pi r \times \frac{dr}{dt} = 4 \times 10^6 \Rightarrow \frac{dr}{dt} = \frac{5 \times 4 \times 10^6}{\pi r}$

$\therefore \frac{dA}{dt} = 2\pi r \times \frac{5 \times 4 \times 10^6}{\pi r} = 4 \times 10^7 \text{ cm}^2/\text{minute}$

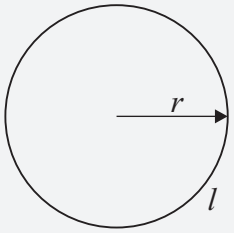
or

$\frac{1}{10}\pi r^2 = (4 \times 10^6)t$

$A = \pi r^2 = (4 \times 10^7)t$

$\therefore \frac{dA}{dt} = 4 \times 10^7 \text{ cm}^2/\text{minute}$

Formulae and Tables Book
Length and area:
Circle [page 8]



$l = 2\pi r$

$A = \pi r^2$

Marking Scheme Notes
Question 8 (c) [Scale 10C (0, 4, 8, 10)] Note: two solutions
1st solution
4: • Mentions relevant rate of change
8: • States chain rule i.e. $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$
or
2nd solution
4: • Effort to establish value of A
8: • A in terms of t
Note: Must use calculus to get any credit

Question 8 (d)

$r = 1 \text{ km} = 10^5 \text{ cm}$

$A = \pi r^2 = \pi(10^5)^5 = 10^{10}\pi \text{ cm}^2$

$\frac{dA}{dt} = 4 \times 10^7 \text{ cm}^2/\text{minute}$

$\text{Time } t = \frac{10^{10}\pi \text{ cm}^2}{4 \times 10^7 \text{ cm}^2/\text{minute}} = 785.4 \text{ minutes} \approx 13 \text{ hours}$

or

$r = 1 \text{ km} = 10^5 \text{ m}$

$\frac{dr}{dt} = \frac{20 \times 10^6}{\pi r}$

$\pi r dr = 20 \times 10^6 dt$

$\int_0^{10^5} \pi r dr = \int_0^t 20 \times 10^6 dt$

$\left[\frac{\pi r^2}{2}\right]_0^{10^5} = [20 \times 10^6 t]_0^t$

Formulae and Tables Book
Calculus: Integrals [page 26]
$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$

$\frac{\pi(10^5)^2}{2} - \frac{\pi(0)^2}{2} = 20 \times 10^6 t$

$10^{10}\pi = 40 \times 10^6 t$

$10^4 \pi = 40t$

$\therefore t = \frac{10^4 \pi}{40} = 250\pi \text{ minutes} = \frac{250\pi}{60} \text{ hours} \approx 13 \text{ hours}$

Marking Scheme Notes
Question 8 (d) [Scale 10C (0, 4, 8, 10)]
4: • r in centimetres
• Effort at expression of area
8: • Correct expression for time