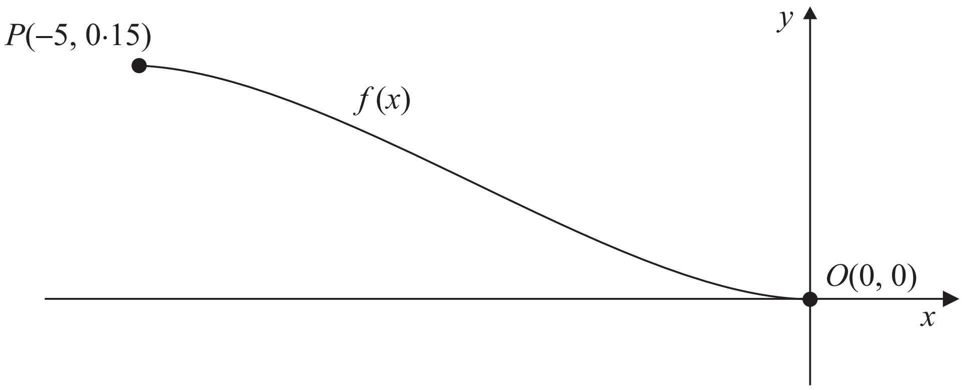
LC 2015: Paper 1

Question 7 (50 marks)

Question 7 (a) (i)



$f(x) = 0\cdot0024x^3 + 0\cdot018x^2 + cx + d \;\leftarrow (0,\,0) \in f(x) \Rightarrow f(0) = 0.$

$f(0) = 0 \Rightarrow 0\cdot0024(0)^3 + 0\cdot018(0)^2 + c(0) + d = 0$

$\therefore d = 0$

Marking Scheme Notes
Question 7 (a) (i) [Scale 5B (0, 2, 5)]
2: • Recognises $x = 0$

Question 7 (a) (ii)

$f(x) = 0\cdot0024x^3 + 0\cdot018x^2 + cx \;\leftarrow (-5,\,0\cdot15) \in f(x) \Rightarrow f(-5) = 0\cdot15.$

$f(-5) = 0\cdot15 \Rightarrow 0\cdot0024(-5)^3 + 0\cdot018(-5)^2 + c(-5) = 0\cdot15$

$0\cdot15 - 5c = 0\cdot15$

$5c = 0$

$\therefore c = 0$

or

The plane land horizontally at O. Therefore $f'(x) = 0$ when $x = 0$.

$f(x) = 0\cdot0024x^3 + 0\cdot018x^2 + cx$

$f'(x) = 0\cdot0072x^2 + 0\cdot036x + c$

$f'(0) = 0 \Rightarrow f'(0) = 0\cdot0072(0)^2 + 0\cdot036(0) + c = 0$

$\therefore c = 0$

Marking Scheme Notes
Question 7 (a) (ii) [Scale 5B (0, 2, 5)] Note: two solutions
1st solution
2: • Uses $x = -5$ or $f(x) = 0\cdot15$
5: • Begins with $c = 0$ and shows $f(-5) = 0\cdot15$ or similar
or
2nd solution
2: • Uses $x = -5$
• Gets $f'(x)$
• Uses $f'(x) = 0$ when $x = 0$

Question 7 (b) (i)

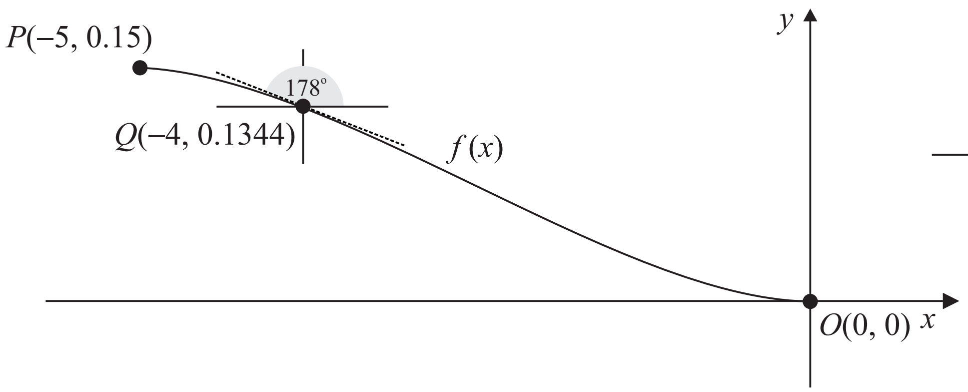
$$f(x) = 0\cdot0024x^3 + 0\cdot018x^2$$
$$f'(x) = 0\cdot0072x^2 + 0\cdot036x$$
$$f'(-4) = 0\cdot0072(-4)^2 + 0\cdot036(-4)$$
$$= -\tfrac{18}{625}$$
$$= -0\cdot0288$$

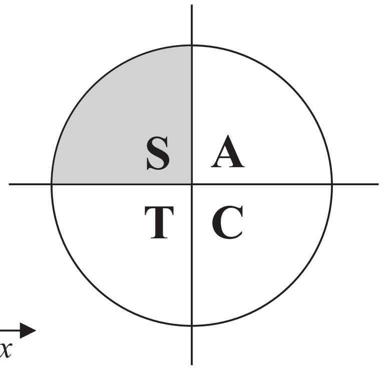
Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$
$$y = [f(x)]^n \Rightarrow \frac{dy}{dx} = n[f(x)]^{n-1} \times f'(x)$$

Marking Scheme Notes
Question 7 (b) (i) [Scale 10C (0, 3, 7, 10)]
3: • Any term correctly differentiated
7: • Correct differentiation
10: • $-\frac{18}{225}$ is a correct answer

Question 7 (b) (ii)





The slope of the tangent to $f(x)$ at $x = -4$ is given by $f'(-4)$. The slope is also the tan the angle makes with the $+x$-axis.

$\tan\theta = -0\cdot0288$ [Second quadrant]

$\alpha = \tan^{-1}(0\cdot0288) = 1\cdot65^\circ$ [Reference angle in first quadrant]

$\theta = 180^\circ - 1\cdot65^\circ \approx 178^\circ$ [Second quadrant]

Angle of descent = 2°

Marking Scheme Notes
Question 7 (b) (ii) [Scale 5B (0, 2, 5)]
2: • Recognition of connection between slope and tan θ
• Any right angled triangle

Question 7 (c)

$$f'(x) = 0\cdot0072x^2 + 0\cdot036x$$
$$f''(x) = 0\cdot0144x + 0\cdot036$$
$$f''(x) = 0 \Rightarrow 0\cdot0144x + 0\cdot036 = 0$$
$$\therefore x = -2\cdot5$$
$$f(x) = 0\cdot0024x^3 + 0\cdot018x^2$$
$$f(-2\cdot5) = 0\cdot0024(-2\cdot5)^3 + 0\cdot018(-2\cdot5)^2 = 0\cdot075$$

Point of inflection $(-2\cdot5, 0\cdot075)$

Point of inflection: Put $\frac{d^2y}{dx^2} = 0$ or $f''(x) = 0$ and solve for x.

Marking Scheme Notes
Question 7 (c) [Scale 10D (0, 2, 5, 8, 10)]
2: • Some correct differentiation of $f'(x)$
• Mention of $f'(x)$
5: • Correct $f''(x)$
8: • Value of x substituted

Question 7 (d) (i)

$$y = 0\cdot0024x^3 + 0\cdot018x^2$$

$$\begin{aligned}
f(-x-5) &= 0\cdot0024(-x-5)^3 + 0\cdot018(-x-5)^2 \\
&= (-x-5)^2[0\cdot0024(-x-5) + 0\cdot018] \\
&= (x^2+10x+25)[-0\cdot0024x - 0\cdot012 + 0\cdot018] \\
&= (x^2+10x+25)[-0\cdot0024x + 0\cdot006] \\
&= -0\cdot0024x^3 + 0\cdot006x^2 - 0\cdot024x^2 + 0\cdot06x - 0\cdot06x + 0\cdot15 \\
&= -0\cdot0024x^3 - 0\cdot018x^2 + 0\cdot15 \\
&= -(0\cdot0024x^3 + 0\cdot018x^2) + 0\cdot15 \\
&= -y + 0\cdot15
\end{aligned}$$

Marking Scheme Notes
Question 7 (d) (i) [Scale 5C (0, 2, 4, 5)]
2: • Some correct substitution
4: • Correct expansions

Question 7 (d) (ii)

$-x-5 \to -2\cdot5\ [x+2.5]$

$-y+0\cdot15 \to 0\cdot075\ [y-0\cdot075]$

$-x-5 \to -2\cdot5 \to x$

$-y+0\cdot15 \to 0\cdot075 \to y$

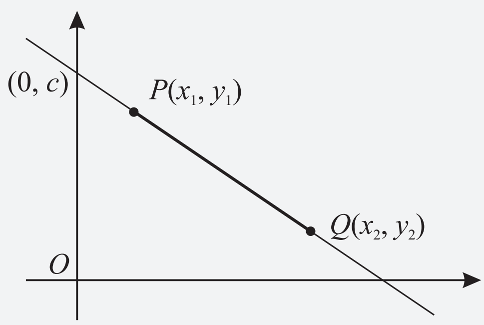
$\therefore (-x-5, -y+0\cdot15) \to (-2\cdot5, 0\cdot075) \to (x, y)$

or

Let (x, y) be the image.
The point of inflection will be the midpoint of $(-x-5, -y+0\cdot15)$ and (x, y).

$$\begin{aligned}
\text{Midpoint} &= \left(\frac{-x-5+x}{2}, \frac{-y+0\cdot15+y}{2}\right) \\
&= (-2\cdot5, 0\cdot075)
\end{aligned}$$

Formulae and Tables Book
Co-ordinate geometry: Line



Midpoint of PQ [page 18]

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Marking Scheme Notes

Question 7 (d) (ii) [Scale 10C (0, 4, 8, 10)] Note: two solutions

1st solution

4: • Work leading to change in x-value or y-value

8: • Correct change in x and y values

or

2nd solution

4: • Uses (x, y) as image, and no more

8: • Effort at calculating mid-point