LC 2015: Paper 1

Question 6 (25 marks)

Question 6 (a)

(i) **Bank A**: Monthly interest rate $r_M = 0{\cdot}35\% \Rightarrow i_M = 0{\cdot}0035$
$(1 + i_M)^{12} = (1 + i_A)$, where i_A is the annual interest rate
$(1 + 0{\cdot}0035)^{12} = (1 + i_A)$
$(1{\cdot}0035)^{12} = (1 + i_A)$
$1{\cdot}042818 = 1 + i_A$
$i_A = 0{\cdot}042818$
$\therefore$ Annual percentage rate (APR) $r_A = 4{\cdot}28\%$ [Given to 3 significant figures]

(ii) **Bank B**: Annual interest rate $r_A = 4{\cdot}5\% \Rightarrow i_A = 0{\cdot}045$
Monthly interest rate: $i_M = ?$, $r_M = ?$
$(1 + i_M)^{12} = (1 + 0{\cdot}045)$
$(1 + i_M)^{12} = (1{\cdot}045)$
$i_M = 1{\cdot}045^{\frac{1}{12}} - 1 = 0{\cdot}0036748$
$\therefore$ $r_M = 0{\cdot}367\%$ [Given to 3 significant figures]

Marking Scheme Notes
Question 6 (a) (i) (ii) [Scale 10C (0, 4, 8, 10)]
4: • Correct formula in either part
• Correct substitution in incorrect formula
8: • Any one section correct
Note: Rate as 0·367% or 0·00367 gets High Partial.

Question 6 (b)

Formulae and Tables Book
Financial mathematics: Amortisation - mortgages and loans
(equal repayments at equal intervals) [page 31]

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

t = Time period (in years)
i = (Annual) rate of interest expressed as a decimal
A = (Annual) repayment amount
P = Principal
Note: The time period can be months or weeks instead of years provided the interest rate is given for that time period.

Method 1: Use the amortisation formula

Time period: Months
$r_M = 0{\cdot}35\% \Rightarrow i_M = 0{\cdot}0035$
$t = 10 \times 12 = 120$ months
$P = €80\,000$

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$
$$= 80\,000\frac{0{\cdot}0035(1{\cdot}0035)^{120}}{(1{\cdot}0035)^{120} - 1}$$
$$= €817{\cdot}59 \approx €818$$

Method 2: Use a geometric series

$$\frac{A}{1\cdot0035}+\frac{A}{1\cdot0035^2}+\ldots+\frac{A}{1\cdot0035^{120}}=80\ 000$$

$$A\left[\frac{1}{1\cdot0035}+\frac{1}{1\cdot0035^2}+\ldots+\frac{1}{1\cdot0035^{120}}\right]=80\ 000$$

$$A\left[\frac{\frac{1}{1\cdot0035}\left(1-\left(\frac{1}{1\cdot0035}\right)^{120}\right)}{1-\frac{1}{1\cdot0035}}\right]=80\ 000$$

$$\therefore A=\frac{80\ 000\left(1-\frac{1}{1\cdot0035}\right)}{\frac{1}{1\cdot0035}\left(1-\left(\frac{1}{1\cdot0035}\right)^{120}\right)}=€817\cdot59\approx€818$$

Formulae and Tables Book
Sequences and series:
Geometric series [page 22]

$$S_n=\frac{a(1-r^n)}{1-r}$$

Marking Scheme Notes
Question 6 (b) [Scale 15C (0, 5, 10, 15)] Note: two solutions
1st solution
5: • Any correct step, i.e. correct formula
10: • Substitution in correct formula
or
2nd solution
5: • Correct equation.
• Listing some terms
• Some substitution
10: • Complete substitution and effort at evaluation.

Note: If A and 80 000 interchanged and remainder of work correct, may get High Partial credit.