LC 2015: Paper 1

Question 5 (25 marks)

Question 5 (a)

$x = \sqrt{x+6}$ ← Square both sides.

$x^2 = x + 6$

$x^2 - x - 6 = 0$ ← Factorise the quadratic.

$(x+2)(x-3) = 0$

$x = \cancel{-2}, 3$

$\therefore x = 3$

Check each solution:

$x = -2$: LHS: $x = -2$

RHS: $\sqrt{x+6} = \sqrt{-2+6} = \sqrt{4} = 2$

Therefore, $x = -2$ is not a solution.

$x = 3$: LHS: $x = 3$

RHS: $\sqrt{x+6} = \sqrt{3+6} = \sqrt{9} = 3$

Therefore, $x = 3$ is a solution.

Marking Scheme Notes

Question 5 (a) [Scale 10C (0, 4, 8, 10)]

4: • Indication of squaring

8: • Correct roots

Note: must indicate required root

Question 5 (b)

$y = x - \sqrt{x+6} = x - (x+6)^{\frac{1}{2}}$

$\frac{dy}{dx} = 1 - \frac{1}{2}(x+6)^{-\frac{1}{2}} = 1 - \frac{1}{2\sqrt{x+6}}$

Formulae and Tables Book

Calculus: Derivatives [page 25]

$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$

$y = [f(x)]^n \Rightarrow \frac{dy}{dx} = n[f(x)]^{n-1} \times f'(x)$

Marking Scheme Notes

Question 5 (b) [Scale 5B (0, 2, 5)]

2: • Any correct differentiation

• Indication of $(x+6)^{\frac{3}{2}}$

Question 5 (c)

$\frac{dy}{dx} = 0 \Rightarrow = 1 - \frac{1}{2\sqrt{x+6}} = 0$

$1 = \frac{1}{2\sqrt{x+6}}$

$2\sqrt{x+6} = 1$ ← Square both sides.

$4(x+6) = 1$

$4x + 24 = 1$

$4x = -23$

$x = -\frac{23}{4}$

$x = -\frac{23}{4}: y = x - \sqrt{x+6} = -\frac{23}{4} - \sqrt{-\frac{23}{4} + 6} = -\frac{23}{4} - \sqrt{\frac{1}{4}} = -\frac{23}{4} - \frac{1}{2} = -\frac{25}{4}$

Turning point $(-\frac{23}{4}, -\frac{25}{4})$

Find turning points (local maxiumum/minimum)

Put $\frac{dy}{dx} = 0$ and solve for x

Marking Scheme Notes
Question 5 (c) [Scale 10C (0, 4, 8, 10)]
4: • Differentiation equals 0
8: • Finds x value

Note 1: A linear equation from $f'(x)$ gets low partial at most
Note 2: Must put $f'(x) = 0$ in (c) to get any marks
Note 3: $f'(x)$ only and $f''(x)$ only: no credit