

LC 2015: PAPER 1

QUESTION 4 (25 MARKS)

Question 4 (a)

$$z_2 = 2 + 3i, z_3 = 3 - 2i$$

$$\begin{aligned} \frac{2}{z_1} &= \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{2+3i} + \frac{1}{3-2i} \\ &= \frac{3-2i+2+3i}{(2+3i)(3-2i)} = \frac{5+i}{12+5i} \end{aligned}$$

$$\begin{aligned} \frac{z_1}{2} &= \frac{12+5i}{5+i} \\ &= \frac{12+5i}{5+i} \times \frac{5-i}{5-i} = \frac{65+13i}{26} = \frac{13(5+i)}{26} = \frac{5+i}{2} \end{aligned}$$

$$\therefore z_1 = 5+i$$

or

$$\frac{1}{z_2} = \frac{1}{2+3i} = \frac{1}{2+3i} \times \frac{2-3i}{2-3i} = \frac{2-3i}{13}$$

$$\frac{1}{z_3} = \frac{1}{3-2i} = \frac{1}{3-2i} \times \frac{3+2i}{3+2i} = \frac{3+2i}{13}$$

$$\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$$

$$\frac{2}{z_1} = \frac{2-3i}{13} + \frac{3+2i}{13} = \frac{2-3i+3+2i}{13} = \frac{5-i}{13}$$

$$\therefore z_1 = \frac{26}{5-i} = \frac{26}{5-i} \times \frac{5+i}{5+i} = \frac{26(5+i)}{26} = 5+i$$

FORMULA: Complex Numbers

Conjugates \bar{z}

$$z = a + bi \Rightarrow \bar{z} = a - bi$$

Multiplying a complex number by its conjugate:

$$z\bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

DIVISION OF COMPLEX NUMBERS

$$\frac{1}{a+bi} = \frac{1}{a+bi} \times \frac{a-bi}{a-bi} = \frac{a-bi}{a^2+b^2}$$

MARKING SCHEME NOTES

Question 4 (a) [Scale 15D (0, 4, 7, 11, 15)]

- 4: • Some rationalisation
• Some relevant rearrangement

- 7: • Gets z_1 or $\frac{2}{z_1}$ in the form of $\frac{a+bi}{c+di}$

- 11: • Correct use of conjugate in $\frac{12+5i}{5+i}$

or

- 4: • One complex number correct

- 7: • Two complex numbers correct

- 11: • Correct use of conjugate in $\frac{2}{a+bi} = \frac{5-i}{13}$

Question 4 (b)

$$S_n = 1 + \omega + \omega^2 + \dots + \omega^{n-1}$$

$$a = 1, r = \omega$$

$$\therefore S_n = \frac{1(1 - \omega^n)}{1 - \omega}$$

$$\omega^n = 1$$

$$\therefore S_n = \frac{1(1 - 1)}{1 - \omega} = 0$$

FORMULAE AND TABLES BOOK**Sequences and series:
Geometric series [page 22]**

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

a is the first term

r is the common ratio

MARKING SCHEME NOTES**Question 4 (b) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- Correct Geometric Progression formula
 - Correct first term
 - Correct ratio
- 8:**
- Values substituted in formula