LC 2015 (Set B): Paper 1

Question 4 (25 marks)

Question 4 (a)

$z_2 = 2 + 3i, z_3 = 3 - 2i$

$$\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{2+3i} + \frac{1}{3-2i}$$

$$= \frac{3-2i+2+3i}{(2+3i)(3-2i)} = \frac{5+i}{12+5i}$$

$$\frac{z_1}{2} = \frac{12+5i}{5+i}$$

$$= \frac{12+5i}{5+i} \times \frac{5-i}{5-i} = \frac{65+13i}{26} = \frac{13(5+i)}{26} = \frac{5+i}{2}$$

$$\therefore z_1 = 5+i$$

Formula: Complex Numbers

Conjugates $\bar{z}$

$z = a+bi \Rightarrow \bar{z} = a-bi$

Multiplying a complex number by its conjugate:

$z\bar{z} = (a+bi)(a-bi) = a^2 + b^2$

Division of complex numbers

$$\frac{1}{a+bi} = \frac{1}{a+bi} \times \frac{a-bi}{a-bi} = \frac{a-bi}{a^2+b^2}$$

or

$$\frac{1}{z_2} = \frac{1}{2+3i} = \frac{1}{2+3i} \times \frac{2-3i}{2-3i} = \frac{2-3i}{13}$$

$$\frac{1}{z_3} = \frac{1}{3-2i} = \frac{1}{3-2i} \times \frac{3+2i}{3+2i} = \frac{3+2i}{13}$$

$$\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$$

$$\frac{2}{z_1} = \frac{2-3i}{13} + \frac{3+2i}{13} = \frac{2-3i+3+2i}{13} = \frac{5-i}{13}$$

$$\therefore z_1 = \frac{26}{5-i} = \frac{26}{5-i} \times \frac{5+i}{5+i} = \frac{26(5+i)}{26} = 5+i$$

Marking Scheme Notes

Question 4 (a) [Scale 15D (0, 4, 7, 11, 15)]

4: • Some rationalisation
• Some relevant rearrangement

7: • Gets z_1 or $\frac{2}{z_1}$ in the form of $\frac{a+bi}{c+di}$

11: • Correct use of conjugate in $\frac{12+5i}{5+i}$

or

4: • One complex number correct

7: • Two complex numbers correct

11: • Correct use of conjugate in $\frac{2}{a+bi} = \frac{5-i}{13}$

Question 4 (b)

$$S_n = 1 + \omega + \omega^2 + ... + \omega^{n-1}$$

$$a = 1,\ r = \omega$$

$$\therefore S_n = \frac{1(1-\omega^n)}{1-\omega}$$

$$\omega^n = 1$$

$$\therefore S_n = \frac{1(1-1)}{1-\omega} = 0$$

Formulae and Tables Book
Sequences and series:
Geometric series [page 22]

$$S_n = \frac{a(1-r^n)}{1-r}$$

a is the first term
r is the common ratio

Marking Scheme Notes
Question 4 (b) [Scale 10C (0, 4, 8, 10)]

4: • Correct Geometric Progression formula
• Correct first term
• Correct ratio

8: • Values substituted in formula