LC 2015: Paper 1

Question 2 (25 marks)

Factor theorem
If k is a root of a polynomial equation $P(x) = 0$, then $(x - k)$ is a factor of $P(x)$ and vice versa.
or
For a polynomial $P(x)$, $P(k) = 0 \Rightarrow P(k) = (x - k)Q(x)$, where $Q(x)$ is a polynomial of degree one less than $P(x)$.

Call the polynomial function $P(x)$. Substitute different integer values of x into this polynomial until you get an answer of 0.
Hint: The only integer values that work are divisors of the constant term in $P(x)$. So try 1, −1, 11 and −11 in order.

$P(x) = x^3 - 3x^2 - 9x + 11$
$P(1) = (1)^3 - 3(1)^2 - 9(1) + 11 = 1 - 3 - 9 + 11 = 0 \leftarrow$ This is successful.
$\therefore (x-1)$ is a factor of $P(x)$

$(x - 1)$ is a linear factor of the cubic polynomial $P(x)$. The other factor will be a quadratic. You can find this quadratic factor by lining up or by division.

Lining up:
Cubic = Linear × Quadratic
$x^3 - 3x^2 - 9x + 11 = (x-1)(x^2 + px - 11)$
$x^3 - 3x^2 - 9x + 11 = x^3 + (p-1)x^2 + (-p-11)x + 11$
Line up $x^2 : -3 = p - 1 \Rightarrow p = -2$

$\therefore P(x) = 0 \Rightarrow x^3 - 3x^2 - 9x + 11 = (x-1)(x^2 - 2x - 11) = 0$

Division:

$$\begin{array}{r|l} & x^2 - 2x - 11 \\ \hline x - 1 & x^3 - 3x^2 - 9x + 11 \\ & \mp x^3 \pm x^2 \\ \hline & \quad -2x^2 - 9x + 11 \\ & \quad \pm 2x^2 \mp 2x \\ \hline & \qquad -11x + 11 \\ & \qquad \pm 11x \mp 11 \\ \hline & \qquad\qquad 0 \end{array}$$

Finally solve the quadratic equation.
$x^2 - 2x - 11 = 0$
$a = 1,\ b = -2,\ c = -11$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4 + 44}}{2} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2}$$
$$= 1 \pm 2\sqrt{3}$$

Answers: $x = 1,\ 1 \pm 2\sqrt{3}$

Formulae and Tables Book
Algebra: Roots of the quadratic equation
$ax^2 + bx + c = 0$ [page 20]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Marking Scheme Notes

Question 2 [Scale 25E (0, 5, 10, 15, 20, 25)]

5: • Effort at finding root, i.e. $f(1), f(-1)$, etc.

10: • Finds one root correctly

• x^2 after division by incorrect factor

• Correct answers in decimal form from calculator with or without work

15: • Tries division and gets x^2 at very minimum

20: • Having got a quadratic equation with no remainder, fills in quadratic formula

• $1 \pm \sqrt{12}$

Note: If there is a remainder after division can only get maximum of 15 marks.