LC 2016 (Set A): Paper 2

Question 9 (50 marks)

Question 9 (a) (i)

Formulae and Tables Book
Statistics and Probability: Probability distribution (standarding formula) [page 34]

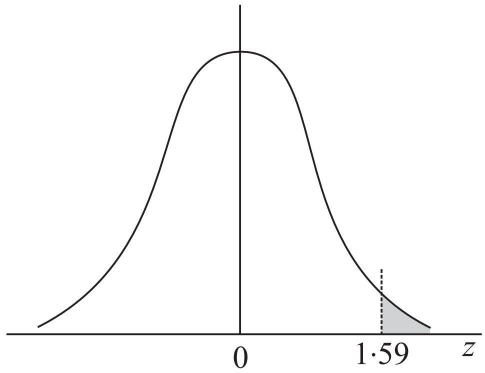
$$z = \frac{x-\mu}{\sigma}$$

n = Number in the sample
σ = standard deviation of the sample

μ = €39 400
σ = €12 920
x = €60 000

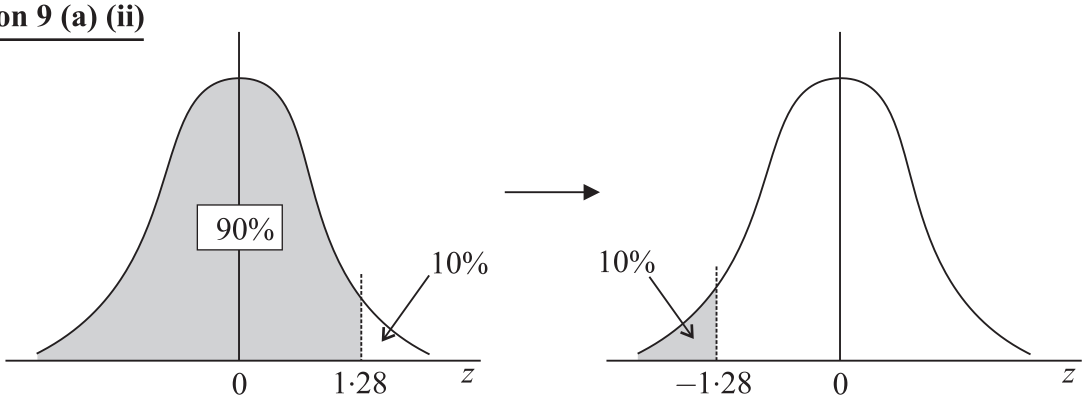
$$z = \frac{x-\mu}{\sigma} = \frac{60\ 000 - 39\ 400}{12\ 920} = 1\cdot 59$$

$$\begin{aligned} P(x \geq 60\ 000) &= P(z \geq 1\cdot 59) \\ &= 1 - P(z \leq 1\cdot 59) \\ &= 1 - 0\cdot 9441 \\ &= 0\cdot 0559 \approx 5\cdot 6\% \end{aligned}$$



Marking Scheme Notes
Question 9 (a) (i) [Scale 10D (0, 3, 5, 8, 10)]
3: • μ and σ identified
5: • $z = 1\cdot 59$
8: • identifies $0\cdot 9441$

Question 9 (a) (ii)



$$P(z \leq Z) = 0\cdot 9 \Rightarrow Z = 1\cdot 28$$
$$\therefore P(z \leq Z) = 0\cdot 1 \Rightarrow Z = -1\cdot 28$$
$$z = \frac{x-\mu}{\sigma} \Rightarrow -1\cdot 28 = \frac{x - 39\ 400}{12\ 920}$$
$$\therefore x = -1\cdot 28 \times 12\ 920 + 39\ 400 \approx €22\ 862$$

Marking Scheme Notes
Question 9 (a) (ii) [Scale 5C (0, 2, 4, 5)]
2: • identifies $1\cdot 28$ but fails to progress
4: • formula for x fully substituted

Question 9 (a) (iii)

$n = 1000$, $\bar{x} = 38\ 280$

$$\bar{Z} = \frac{38\ 280 - 39\ 400}{\frac{12\ 290}{\sqrt{1000}}} = -2\cdot 74$$

$$\bar{z} = \frac{\bar{x} - \bar{\mu}}{\bar{\sigma}}, \text{ where } \bar{\mu} = \mu \text{ and } \bar{\sigma} = \frac{\sigma}{\sqrt{n}}.$$

Null hypothesis H_0: There is no change in the mean annual income.
Alternative hypothesis H_1: There is a change in the mean annual income.

$\bar{Z} = -2\cdot 74 < -1\cdot 96$: Reject the null hypothesis

The result is significant. The mean annual income has changed.

MARKING SCHEME NOTES
Question 9 (a) (iii) [Scale 15D (0, 4, 7, 11, 15)]
4: • z formulated with some substitution
• states null and/or alternative hypothesis only
• reference to $1\cdot96$
7: • z fully substituted
11: • $z = -2\cdot74$ and stops
• fails to state the null and alternative hypothesis correctly
• fails to contextualise the answer

Question 9 (b)

At a 95% confidence level, $\bar{x} - 1\cdot96 \times \frac{\sigma}{\sqrt{n}} \leftarrow \mu \rightarrow \bar{x} + 1\cdot96 \times \frac{\sigma}{\sqrt{n}}$ is the best estimate for μ, where $\bar{x}$ is the mean of the sample and σ is the standard deviation of the population or the sample and is the 95% confidence interval for μ.

$n = 400$, $\bar{x} = 26\ 974$, $\sigma = 5120$

$$\bar{x} - 1\cdot96 \times \frac{\sigma}{\sqrt{n}} \leftarrow \mu \rightarrow \bar{x} + 1\cdot96 \times \frac{\sigma}{\sqrt{n}}$$

$$26\ 974 - 1\cdot96 \times \frac{5120}{\sqrt{400}} \leftarrow \mu \rightarrow 26\ 974 + 1\cdot96 \times \frac{5120}{\sqrt{400}}$$

$$26\ 472\cdot24 \leftarrow \mu \rightarrow 27\ 475\cdot76$$

Confidence interval $= [€26\ 472\cdot24,\ €27\ 475\cdot76]$

MARKING SCHEME NOTES
Question 9 (b) [Scale 10C (0, 3, 7, 10)]
3: • interval formulated with some correct substitution
7: • interval formulated with fully correct substitution

Question 9 (c)

The central limit theorem states that the sampling distribution of any statistic will be normal or nearly normal, if the sample size is large enough, allowing you to find important information like the mean farm size.

MARKING SCHEME NOTES
Question 9 (c) [Scale 5B (0, 2, 5)]
2: • mentions 30 (or more) but not contextualised

Question 9 (d)

$$\text{Margin of error} = 0\cdot045 = \frac{0\cdot98}{\sqrt{n}}$$

$$\sqrt{n} = \frac{0\cdot98}{0\cdot045} \Rightarrow n = \left(\frac{0\cdot98}{0\cdot045}\right)^2 \approx 474$$

MARKING SCHEME NOTES
Question 9 (d) [Scale 5C (0, 2, 4, 5)]
2: • $\frac{1}{\sqrt{n}}$
4: • n formulated with fully correct substitution
NOTE: Accept 493 farmers or 494 farmers

Author's note:
Our solution is different to the solution given in the marking scheme.
Question 9 (d) is clearly a problem on population proportions. We have no estimate for $\widehat{p}$ to work out $\text{ME} = 1\cdot 96\sqrt{\dfrac{\widehat{p}(1-\widehat{p})}{n}}$. In this case we assume $\widehat{p} = 0\cdot 5$ which is the value which maximises $\widehat{p}(1-\widehat{p})$. Therefore, the margin of error is maximised at $\text{ME} = 1\cdot 96\sqrt{\dfrac{0\cdot 5(1-0\cdot 5)}{n}} = \dfrac{0\cdot 98}{\sqrt{n}}$.