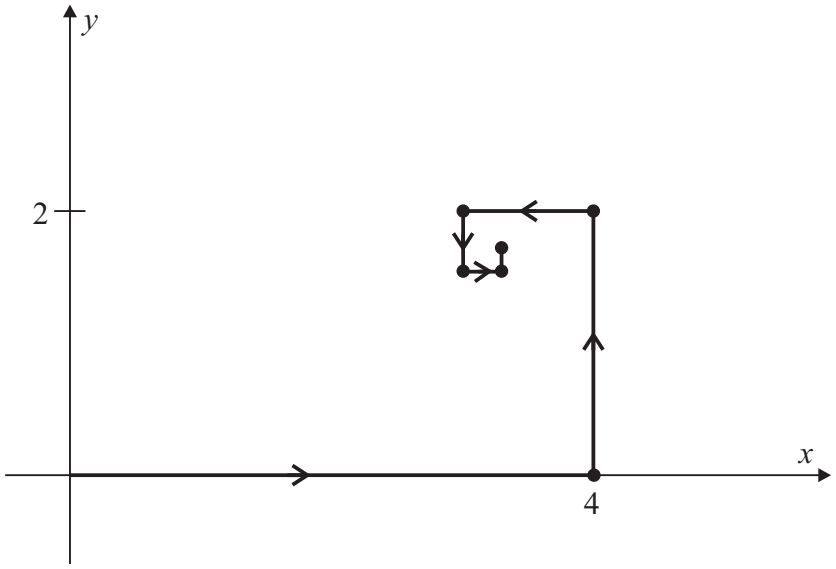
LC 2016 (Set A): Paper 1

Question 9 (55 marks)

Question 9 (a)



Question 9 (a) (i)

$T_1 = 4,\ T_2 = 2,\ T_3 = 1$

$a = 4,\ r = \frac{1}{2}$

$$S_n = 7 \cdot 9375 = \frac{4(1-(\frac{1}{2})^n)}{1-\frac{1}{2}}$$

$$8(1-(\tfrac{1}{2})^n) = 7 \cdot 9375$$

$$1-(\tfrac{1}{2})^n = \frac{7 \cdot 9374}{8} = \frac{127}{128}$$

$$\frac{1}{2^n} = 1 - \frac{127}{128} = \frac{1}{128} = \frac{1}{2^7}$$

$\therefore n = 7$

Formulae and Tables Book
Sequences and series:
Geometric series [page 22]

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

Question 9 (a) (ii)

$$S_\infty = \frac{4}{1-\frac{1}{2}} = 8 \text{ units}$$

Question 9 (a) (iii)

Stage	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Change in *x*	+4	0	-1	0	$\frac{1}{4}$	0	$-\frac{1}{16}$	0	$\frac{1}{64}$
Change in *y*	0	+2	0	$-\frac{1}{2}$	0	$\frac{1}{8}$	0	$-\frac{1}{32}$	0

x sequence: $4, -1, \frac{1}{4}, \ldots$

$a = 4,\ r = -\frac{1}{4}$

$$S_\infty = \frac{4}{1-(-\frac{1}{4})} = \frac{4}{\frac{5}{4}} = \frac{16}{5} = 3 \cdot 2$$

y sequence: $2, -\frac{1}{2}, \frac{1}{8}, \ldots$

$a = 2,\ r = -\frac{1}{4}$

$$S_\infty = \frac{2}{1-(-\frac{1}{4})} = \frac{2}{\frac{5}{4}} = \frac{8}{5} = 1 \cdot 6$$

Final point = (3·2, 1·6)

Marking Scheme Notes

Question 9 (a) (i) [Scale 10C (0, 3, 7, 10)]

3: • some listing of terms
• S_n formula

7: • listing of exactly 7 correct terms
• formula fully substituted

Question 9 (a) (ii) [Scale 10C (0, 3, 7, 10)]

3: • S_∞ formula

7: • formula fully substituted

Question 9 (a) (iii) [Scale 15D (0, 4, 7, 11, 15)]

4: • 2 extra entries correct in either row

7: • either row fully correct

11: • one co-ordinate correct

Notes:
- need to see S_∞ correctly used to move beyond Mid Partial Credit
- no S_∞ merits Mid Partial Credit at most

Question 9 (b) (i)

G_1	G_2	G_3	G_4	G_5
			Female	
		Female		
Male	→ Female		Male	
		Male	→ Female	

Question 9 (b) (ii)

$G_{n+2} = G_{n+1} + G_n$

$G_1 = 1$

$G_2 = 1$

$G_3 = G_2 + G_1 = 1 + 1 = 2$

$G_4 = G_3 + G_2 = 2 + 1 = 3$

$G_5 = G_4 + G_3 = 3 + 2 = 5$

$G_6 = G_5 + G_4 = 1 + 1 = 5 + 3 = 8$

$G_7 = G_6 + G_5 = 8 + 5 = 13$

Question 9 (b) (iii)

Method 1

$$G_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

$$G_3 = \frac{(1+\sqrt{5})^3 - (1-\sqrt{5})^3}{2^3\sqrt{5}} = 2 \text{ [Using calculator]}$$

Method 2

$$G_3 = \frac{(1+\sqrt{5})^3 - (1-\sqrt{5})^3}{2^3\sqrt{5}} \text{ [Difference of two cubes]}$$

$$= \frac{(1+\sqrt{5}-1+\sqrt{5})((1+\sqrt{5})^2 + (1+\sqrt{5})(1-\sqrt{5}) + (1-\sqrt{5})^2)}{8\sqrt{5}}$$

$$= \frac{(2\sqrt{5})(1+2\sqrt{5}+5+1-5+1-2\sqrt{5}+5)}{8\sqrt{5}}$$

$$= \frac{8}{4} = 2$$

Marking Scheme Notes

Question 9 (b) (i) [Scale 5B (0, 2, 5)]

2: • one correct entry

Question 9 (b) (ii) [Scale 10C (0, 3, 7, 10)]

3: • $G_6 = G_5 + G_4$
• $G_7 = G_6 + G_5$
• G_7 or G_6 correct
• 8 and/or 13 without work

7: • correct substitution in both

Question 9 (b) (iii) [Scale 5B (0, 2, 5)]

2: • some correct substitution
• using approximate value for $\sqrt{5}$
• $G_3 = 2$
• some effort at cubing

Note: use of $\sqrt{5}$ as approximation, even if rounded off to 2 at end of work merits at most Partial Credit