

LC 2016 (SET A): PAPER 1

QUESTION 8 (55 MARKS)

Question 8 (a) (i)

$$f(x) = -0.274x^2 + 1.193x + 3.23$$

$$A(-0.5, 2.565), B(4.5, 3.05)$$

FORMULAE AND TABLES BOOK
Calculus: Derivatives [page 25]

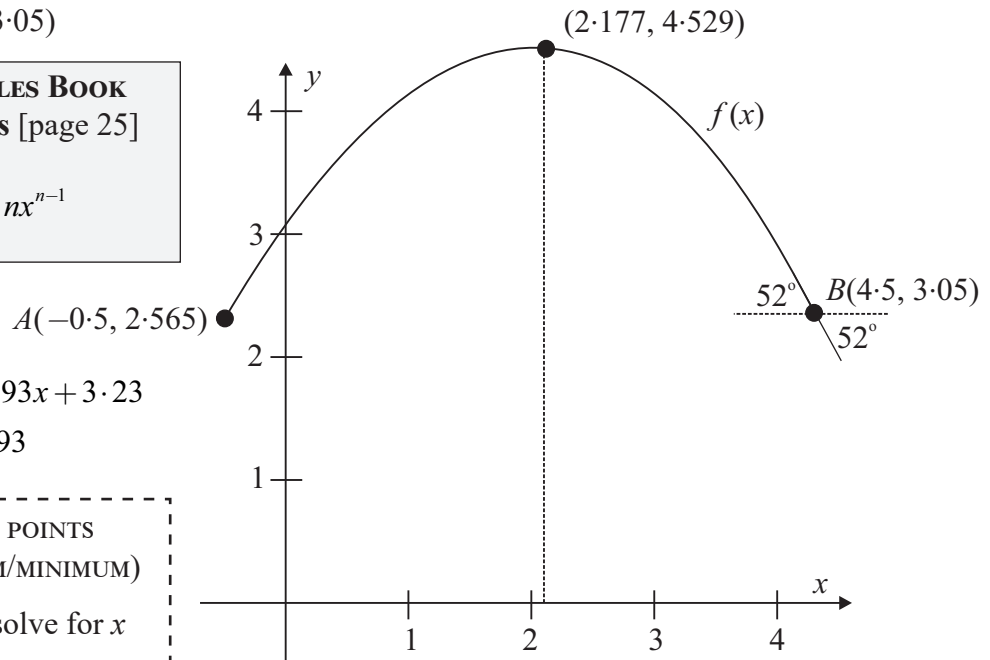
$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

$$f(x) = -0.274x^2 + 1.193x + 3.23$$

$$f'(x) = -0.548x + 1.193$$

FIND TURNING POINTS
(LOCAL MAXIMUM/MINIMUM)

Put $\frac{dy}{dx} = 0$ and solve for x



Maximum:

$$f'(x) = 0 \Rightarrow -0.548x + 1.193 = 0$$

$$0.548x = 1.193$$

$$\therefore x = \frac{1.193}{0.548} = 2.177 \text{ m}$$

$$\begin{aligned} f(2.177) &= -0.274(2.177)^2 + 1.193(2.177) + 3.23 \\ &= 4.529 \text{ m} \end{aligned}$$

The maximum height reached by the ball is 4.529 m.

MARKING SCHEME NOTES

Question 8 (a) (i) [Scale 10C (0, 3, 7, 10)]

3:

- any correct differentiation
- effort made at completing square
- trial and error with more than one value of x tested

7:

- x value correct

NOTE: if correct answer by trial and error, must show points on each side of max point to be lower to earn full credit

Question 8 (a) (ii)

Basket $B(4.5, 3.05)$

$$f'(x) = -0.548x + 1.193$$

$$f'(4.5) = -0.548(4.5) + 1.193 = -1.273 = \text{Slope } m = \tan \theta$$

$$\text{Acute angle} = \tan^{-1}|-1.273| \approx 52^\circ$$

MARKING SCHEME NOTES

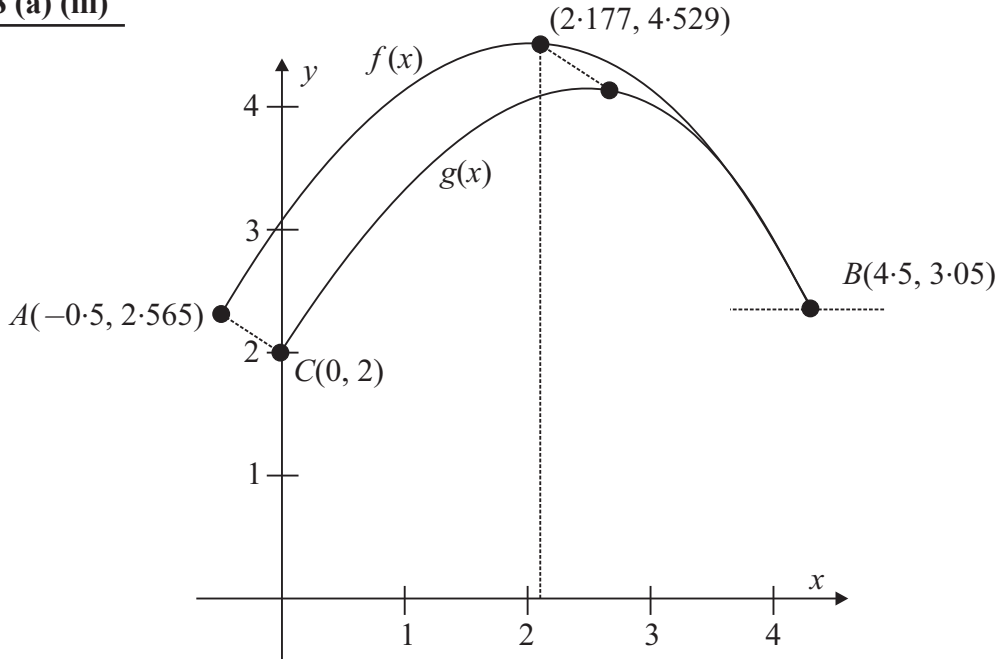
Question 8 (a) (ii) [Scale 5B (0, 2, 5)]

2:

- \tan

NOTE: right angled triangles may appear in diagram given in equation

Question 8 (a) (iii)



$$A(-0.5, 2.565) \rightarrow C(0, 2) [+0.5, -0.565]$$

$$(2.177, 4.529) \rightarrow (2.177 + 0.5, 4.529 - 0.565) = (2.677, 3.964)$$

Question 8 (a) (iv)

VERTICAL AND HORIZONTAL SHIFTS

If $y = f(x)$, then:

1. $f(x) + v$ shifts $f(x)$ vertically up by v units, if $v > 0$.
2. $f(x) - v$ shifts $f(x)$ vertically down by v units, if $v > 0$.
3. $f(x + h)$ shifts $f(x)$ horizontally left by h units, if $h > 0$.
4. $f(x - h)$ shifts $f(x)$ horizontally right by h units, if $h > 0$.

$f(x)$ is shifted right and down: $h = 0.5, v = 0.565$

$$f(x) = -0.274x^2 + 1.193x + 3.23$$

$$g(x) = f(x - h) - v = f(x - 0.5) - 0.565$$

$$= -0.274(x - 0.5)^2 + 1.193(x - 0.5) + 3.23 - 0.565$$

$$= -0.274(x^2 - x + 0.25) + 1.193(x - 0.5) + 3.23 - 0.565$$

$$= -0.274x^2 + 0.274x - 0.0685 + 1.193x - 0.5965 + 3.23 - 0.565$$

$$= -0.274x^2 + 1.467x + 2$$

MARKING SCHEME NOTES

Question 8 (a) (iii) [Scale 5B (0, 2, 5)]

2: • $(-0.5, 2.565) \rightarrow (0, 2)$

Question 8 (a) (iv) [Scale 10D (0, 2, 5, 8, 10)]

2: • equation of curve

• use of C

5: • using peak value

8: • value of a found

Question 8 (b)

Event	x	Formula	a	b	c
200 m race	Time (s)	$y = a(b - x)^c$	4.99087	42.5	1.81
Javelin	Distance (m)	$y = a(x - b)^c$	15.9803	3.8	1.04

x = Time taken (s) or distance achieved (m)

y = Points scored

Question 8 (b) (i)

200 m

$$x = 23.8 \text{ s}$$

$$y = a(b - x)^c = 4.99087(42.5 - 23.8)^{1.81}$$

$$\approx 1000 \text{ points}$$

Javelin

$$x = 58.2 \text{ m}$$

$$y = a(x - b)^c = 15.9803(58.2 - 3.8)^{1.04}$$

$$\approx 1020 \text{ points}$$

Question 8 (b) (ii)

Javelin

$$y = 1295 \text{ points}$$

$$y = a(x - b)^c$$

$$1295 = 15.9803(x - 3.8)^{1.04}$$

$$\frac{1295}{15.9803} = (x - 3.8)^{1.04}$$

$$\left(\frac{1295}{15.9803}\right)^{\frac{1}{1.04}} = x - 3.8$$

$$\therefore x = \left(\frac{1295}{15.9803}\right)^{\frac{1}{1.04}} + 3.8 = 72.23 \text{ m}$$

Question 8 (b) (iii)

Event	x	Formula	a	b	c
200 m race	Time (s)	$y = a(b - x)^c$	4.99087	42.5	1.81
800 m race	Distance (m)	$y = a(b - x)^c$	0.11193	254	?

800 m: $x = 2$ minutes and $1.84 \text{ s} = 121.84 \text{ s}$, $y = 1087$ points

$$y = a(b - x)^c$$

$$1087 = 0.11193(254 - 121.84)^c$$

$$\frac{1087}{0.11193} = 132.16^c$$

$$9711.43 = 132.16^c$$

$$\therefore c = \frac{\log_{10} 9711.43}{\log_{10} 132.16} \approx 1.88$$

MARKING SCHEME NOTES

Question 8 (b) (i) [Scale 10D (0, 2, 5, 8, 10)]

2: • some relevant substitution into one formula

5: • one value of y found

• some relevant substitution into both formulas

8: • one value correct and some relevant substitution into second formula

• uses incorrect formula (once only)

Question 8 (b) (ii) [Scale 5B (0, 2, 5)]

2: • some relevant substitution into formula

Question 8 (b) (iii) [Scale 10C (0, 3, 7, 10)]

3: • some relevant substitution into formula

7: • fully correct substitution into formula