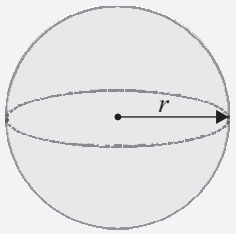
LC 2016 (Set A): Paper 1

Question 7 (40 marks)

Formulae and Tables Book
Surface area and volume:
Sphere [page 8]



$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Question 7 (a) (i)

$\frac{dV}{dt} = 250 \text{ cm}^3/\text{s}$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2 = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 250 = 4\pi r^2 \times \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{250}{4\pi r^2}$

$r = 20 : \frac{dr}{dt} = \frac{250}{4\pi(20)^2} = \frac{5}{32\pi}$ cm/s

Marking Scheme Notes
Question 7 (a) (i) [Scale 10C (0, 3, 7, 10)]

3: • work towards $\frac{dv}{dr}$ or $\frac{dv}{dt}$ or $\frac{dr}{dt}$

7: • correct expression for $\frac{dr}{dt}$

Question 7 (a) (ii)

$A = 4\pi r^2 \Rightarrow \frac{dA}{dr} = 8\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 8\pi r \times \frac{dr}{dt}$

$r = 20 : \frac{dA}{dt} = 8\pi(20) \times \frac{5}{32\pi} = 25 \text{ cm}^2/\text{s}$

Marking Scheme Notes
Question 7 (a) (ii) [Scale 10C (0, 3, 7, 10)]

3: • work towards $\frac{dA}{dr}$ or $\frac{dA}{dt}$

7: • correct expression for $\frac{dA}{dt}$

Question 7 (b) (i)

$f(x) = -x^2 + 10x$

$f(x) = 0 \Rightarrow -x^2 + 10x = 0$

$x^2 - 10x = 0$

$x(x - 10) = 0$

$\therefore x = 0 \text{ m}, 10 \text{ m}$

Marking Scheme Notes
Question 7 (b) (i) [Scale 10C (0, 3, 7, 10)]

3: • quadratic equation formed
• gets $x = 0$ only

7: • quadratic factorised

Note: $f'(x) = 0 \Rightarrow 2x - 10 = 0 \Rightarrow x = 5$ merits 0 marks

Question 7 (b) (ii) [Scale 10C (0, 3, 7, 10)]

3: • integration set up

7: • correct integration with some substitution

Question 7 (b) (ii)

The average value $\bar{y}$ of a continuous function $y = f(x)$ on an interval $[a, b]$ is given by:

$$\bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{b-a}\int_a^b y\,dx$$

Average $f(x)$

$= \frac{1}{10-0}\int_0^{10} (-x^2 + 10x)\,dx$

$= \frac{1}{10}\left[-\frac{x^3}{3} + \frac{10x^2}{2}\right]_0^{10}$

$= \frac{1}{10}\left[-\frac{x^3}{3} + 5x^2\right]_0^{10}$

$= \frac{1}{10}\left[\left(-\frac{(10)^3}{3} + 5(10)^2\right) - \left(-\frac{(0)^3}{3} + 5(0)^2\right)\right]$

$= \frac{1}{10}\left[-\frac{1000}{3} + 500\right] = \frac{50}{3}$ m