

LC 2016 (SET A): PAPER 1

QUESTION 1 (25 MARKS)

Question 1 (a)

Use the conjugate root theorem to find the other root. Therefore, if z_1 is one of the roots, the conjugate of z_1 is also a root. Call this root z_2 .

$$z_1 = -4 + 3i \Rightarrow z_2 = -4 - 3i$$

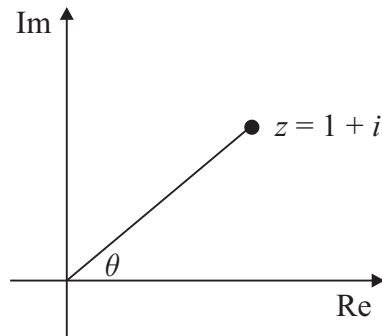
Question 1 (b)

$$z = 1 + i$$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|\tan \theta| = \left| \frac{1}{1} \right| = 1 \Rightarrow \theta = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\therefore \theta = \frac{\pi}{4} \text{ [1st quadrant]}$$



$$z = 1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$\begin{aligned} z^8 &= (\sqrt{2})^8 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^8 \\ &= 2^4 (\cos 2\pi + i \sin 2\pi) \\ &= 16(1 + 0i) = 16 \end{aligned}$$

FORMULAE: Complex Numbers

Modulus $|z|$: $|z| = |a + bi| = \sqrt{a^2 + b^2}$

$z = a + bi$ (Cartesian form)

$z = r(\cos \theta + i \sin \theta)$ (Polar form)

DE MOIVRE'S THEOREM (page 20)

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

MARKING SCHEME NOTES

Question 1 (a) [Scale 5B (0, 2, 5)]

2: • real or imaginary part correct

Question 1 (b) [Scale 10C (0, 3, 7, 10)]

3: • correct answer without use of De Moivre's

• modulus or argument correct

• formula

• statement of de Moivre's

7: • $16(\cos 2\pi + i \sin 2\pi)$

NOTE: not De Moivre and incorrect answer merits 0 marks

Question 1 (c)**METHOD 1**

Forming a quadratic equation from its roots.

$$z^2 - Sz + P = 0$$

S: Sum of the roots**P:** Product of the roots

$$z^2 + (-2+i)z + 3-i = 0$$

Roots: $1+i$, α

$$\text{Sum} = \alpha + 1 + i = -(-2+i)$$

$$\alpha + 1 + i = 2 - i$$

$$\therefore \alpha = 1 - 2i$$

METHOD 2

$$z^2 + (-2+i)z + 3-i = 0$$

$$a = 1, b = (-2+i), c = (3-i)$$

$$z = \frac{-(-2+i) \pm \sqrt{(-2+i)^2 - 4(1)(3-i)}}{2}$$

$$= \frac{2-i \pm \sqrt{4-4i+i^2-12+4i}}{2}$$

$$= \frac{2-i \pm \sqrt{4-4i-1-12+4i}}{2} = \frac{2-i \pm \sqrt{-9}}{2}$$

$$= \frac{2-i \pm 3i}{2} = \frac{2+2i}{2}, \frac{2-4i}{2}$$

$$= 1+i, 1-2i$$

Other root = $(1-2i)$ **MARKING SCHEME NOTES****Question 1 (c) [Scale 10C (0, 3, 7, 10)]**

3: • root formula with some substitution

7: • formula fully substituted

Or

3: • equation rearranged

- $-\frac{b}{a}$

7: • correct substitution