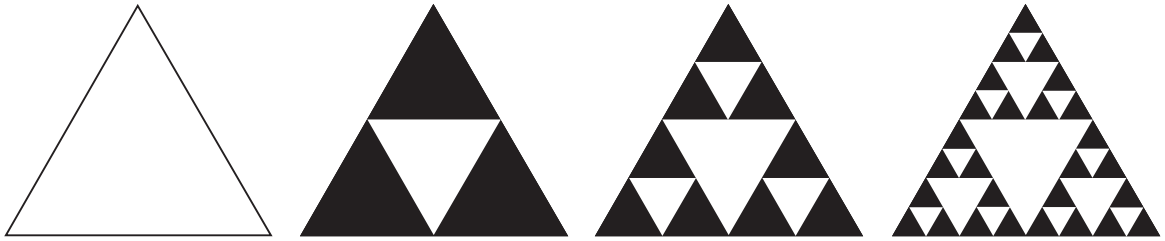
LC 2018: Paper 1

Question 9 (55 marks)



Step 0 Step 1 Step 2 Step 3

Question 9 (a)

Step	0	1	2	3
Number of black triangles	1	3	9	27
Fraction of the original triangle remaining	1	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$

Marking Scheme Notes
Question 9 (a) [Scale 10C (0, 4, 8, 10)]
4: • One correct entry
8: • Three correct entries
Full credit –1: • Answers as decimals

Question 9 (b)

(i) Number of black triangles: 1, 3, 9, 27,...

$n = 0$: Number of black triangles $= 1 = 3^0$

$n = 1$: Number of black triangles $= 3 = 3^1$

$n = 2$: Number of black triangles $= 9 = 3^2$

$n = 3$: Number of black triangles $= 27 = 3^3$

$n = n$: Number of black triangles $= 3^n$

Or

Formula: $T_n = ar^{n-1}$

$[n = 1$ is first term in formula$]$

Sequence: 3, 9, 27,...

$a = 3,\ r = 3$

$T_n = 3 \times 3^{n-1} = 3^n$

(ii) $3^k > 1 \times 10^9$

$k > \log_3(1 \times 10^9) = 18 \cdot 9$

$\therefore k$ is the 19^{th} step.

Marking Scheme Notes
Question 9 (b) (i) [Scale 5B (0, 2, 5)]
2: • $3n$ written
• n^3 written
Full credit -1: • 3^{n-1} written

Question 9 (b) (ii) [Scale 5C (0, 3, 4, 5)]
3: • $3^n > 1\ 000\ 000\ 000$
4: • Inequality with k not written as an index
Note: if $3k$ or k^3 from above used fully here then award low partial credit at most

Question 9 (c)

(i) Fraction remaining: $1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \ldots$

$n = 0$: Fraction remaining $= 1 = (\frac{3}{4})^0$

$n = 1$: Fraction remaining $= \frac{3}{4} = (\frac{3}{4})^1$

$n = 2$: Fraction remaining $= \frac{9}{16} = (\frac{3}{4})^2$

$n = 3$: Fraction remaining $= \frac{27}{64} = (\frac{3}{4})^3$

$n = n$: Fraction remaining $= (\frac{3}{4})^n$

Or

Formula: $T_n = ar^{n-1}$

[$n = 1$ is first term in formula]

Sequence: $\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \ldots$ [$a = \frac{3}{4}, r = \frac{3}{4}$]

$T_n = \frac{3}{4} \times (\frac{3}{4})^{n-1} = (\frac{3}{4})^n$

$(\frac{3}{4})^h < \frac{1}{100} \Rightarrow h < \log_{\frac{3}{4}}(\frac{1}{100}) = 16 \cdot 01$

$\therefore h = 17$

(ii) $\lim_{n \to \infty} (\frac{3}{4})^n = 0$

Marking Scheme Notes

Question 9 (c) (i) [Scale 10C (0, 4, 8, 10)]

4: • Correct answer without work
- $(\frac{3}{4})^h$ or candidates ratio to the power of h
- $r = \frac{3}{4}$
- Lists two or more terms

8: • Inequality with h not written as an index

Full credit -1: • $(\frac{3}{4})^{h-1} < \frac{1}{100}$ and finishes correctly

Question 9 (c) (ii) [Scale 5B (0, 2, 5)]

2: • $\lim_{n \to \infty}$
- Some use of $\frac{3}{4}$

5: • Correct answer without work
- $\frac{1}{\infty}$ or equivalent

Question 9 (d)

(i) Table:

Step	0	1	2	3	4
Perimeter	3	$\frac{9}{2}$	$\frac{27}{4}$	$\frac{81}{8}$	$\frac{243}{16}$

(ii) Perimeter: $3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \frac{243}{16}, \ldots$

$n = 0$: Perimeter $= 3 = \frac{3^1}{2^0}$

$n = 1$: Perimeter $= \frac{9}{2} = \frac{3^2}{2^1}$

$n = 2$: Perimeter $= \frac{27}{4} = \frac{3^3}{2^2}$

$n = 3$: Perimeter $= \frac{81}{8} = \frac{3^4}{2^3}$

$n = 4$: Perimeter $= \frac{243}{16} = \frac{3^5}{2^4}$

$n = n$: Perimeter $= \frac{3^{n+1}}{2^n}$

Or Formula: $T_n = ar^{n-1}$

Sequence: $\frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \frac{243}{16}, \ldots$

$a = \frac{9}{2}$, $r = \frac{3}{2}$

$T_n = \frac{9}{2} \times (\frac{3}{2})^{n-1} \Rightarrow T_{35} = \frac{9}{2} \times (\frac{3}{2})^{34} = 4\ 368\ 329$

(iii) As $n \to \infty$, Area $\to 0$

As $n \to \infty$, Perimeter $\to \infty$

Marking Scheme Notes

Question 9 (d) (i) [Scale 10C (0, 4, 8, 10)]

4: • One correct entry
• All numerators correct with all incorrect denominators
• All denominators correct with all incorrect numerators

8: • Two correct entries

Question 9 (d) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Pattern identified
• Recognises $r = \frac{3}{2}$
• Some relevant substitution into
• $T_n = ar^{n-1}$
• $a = 3$ or $a = 4{\cdot}5$

4: • Step 35 $= \frac{3^{36}}{2^{35}}$ or equivalent

Full credit -1: • $T_{35} = (3)\left(\frac{3}{2}\right)^{34}$

Question 9 (d) (iii) [Scale 5C (0, 3, 4, 5)]

3: • $\lim_{n\to\infty}\left(\frac{3^{n+1}}{2^n}\right)$ or equivalent
• Area is getting smaller
• Perimeter is increasing

4: • Area approaches 0
• **Perimeter** $\to \infty$ identified
• Area is getting smaller and Perimeter is increasing