LC 2018: Paper 1

Question 3 (25 marks)

Question 3 (a)

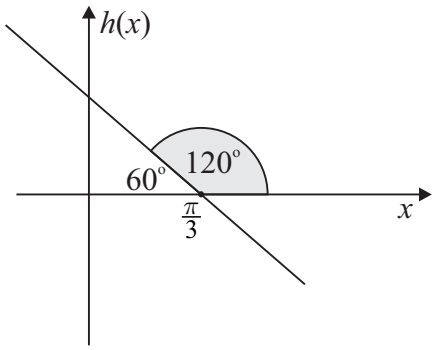
$y = h(x) = \cos(2x)$

$\frac{dy}{dx} = -2\sin 2x$

$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} = -2\sin 2(\frac{\pi}{3}) = -\sqrt{3}$

Slope $m = \tan\theta = -\sqrt{3}$

$\therefore \theta = 120^\circ$



Marking Scheme Notes
Question 3 (a) [Scale 10D (0, 3, 5, 8, 10)]
3: • Differentiation indicated
• Use of 2
5: • Derivative found
8: • tan θ = evaluated derivative
• $\theta = -60°$
Note: Must use differentiation to gain any credit
Note: If integration symbol appears then 0 credit

Question 3 (b)

$y = h(x) = \cos(2x),\ 0 \le x \le \frac{\pi}{4}$

$$\overline{y} = \frac{\int_0^{\frac{\pi}{4}} \cos(2x)\,dx}{\frac{\pi}{4} - 0} = \frac{\frac{1}{2}[\sin 2x]_0^{\frac{\pi}{4}}}{\frac{\pi}{4}} = \frac{2}{\pi}[\sin\tfrac{\pi}{2} - \sin 0] = \frac{2}{\pi}$$

The average value $\overline{y}$ of a continuous function $y = g(x)$ on an interval $[a, b]$ is given by:

$$\overline{y} = \frac{\int_a^b g(x)\,dx}{b-a} = \frac{\int_a^b y\,dx}{b-a}$$

Marking Scheme Notes
Question 3 (b) [Scale 15D (0, 5, 7, 11, 15)]
5: • Integration indicated
7: • cos 2x integrated correctly $\left(\frac{\sin(2x)}{2}\right)$
• $-2\sin 2x$ and finishes correctly
11: • Substitutes limits into integral and stops
• Integral evaluated at $x = \frac{\pi}{4}$ $\left(\text{i.e. omits } \frac{1}{\frac{\pi}{4} - 0}\right)$ and finishes
Note: Errors in integration could include:
An error in the trig function (including sign)
An error in the angle
An error in the application of the chain rule
Note: Must have integration to gain any credit