

LC 2018: PAPER 1

QUESTION 2 (25 MARKS)

Question 2 (a)

Geometric series: $x^2, 5x - 8, x + 8$

$$r = \frac{x+8}{5x-8} = \frac{5x-8}{x^2}$$

$$x^2(x+8) = (5x-8)^2$$

$$x^3 + 8x^2 = 25x^2 - 80x + 64$$

$$x^3 - 17x^2 + 80x - 64 = 0$$

MARKING SCHEME NOTES

Question 2 (a) [Scale 10C (0, 4, 8, 10)]

0: • Treats as an arithmetic sequence

4: • $\frac{5x-8}{x^2}$ or $\frac{x+8}{5x-8}$

• Some effort at finding x in a geometric sequence (must use at least one of the terms)

• $r = \frac{T_n}{T_{n-1}}$ or similar

8: • $\frac{5x-8}{x^2} = \frac{x+8}{5x-8}$

• $(5x-8)^2$ and $x^2(x+8)$

Question 2 (b)

$$f(x) = x^3 - 17x^2 + 80x - 64$$

$$f(1) = (1)^3 - 17(1)^2 + 80(1) - 64 = 1 - 17 + 80 - 64 = 0$$

1 is a root $\Rightarrow (x-1)$ is a factor

$$\begin{aligned} x^3 - 17x^2 + 80x - 64 &= (x-1)(x^2 + kx + 64) \\ &= x^3 + (k-1)x^2 + (64-k)x - 64 \end{aligned}$$

$$k-1 = -17 \Rightarrow k = -16$$

$$x^3 - 17x^2 + 80x - 64 = (x-1)(x^2 - 16x + 64) = 0$$

$$(x-1)(x-8)(x-8) = 0 \Rightarrow x = 1, 8, 8$$

Answer: $x = 8$

or

$$\begin{array}{r} x^2 - 16x + 64 \\ x-1 \overline{) x^3 - 17x^2 + 80x - 64} \\ \underline{\mp x^3 \pm 1x^2} \\ -16x^2 + 80x - 64 \\ \underline{\pm 16x^2 \mp 16x} \\ 64x - 64 \\ \underline{\mp 64x \pm 64} \\ 0 \end{array}$$

MARKING SCHEME NOTES

Question 2 (b) [Scale 10C (0, 4, 8, 10)]

4: • Shows $f(1) = 0$
• Any correct substitution

8: • Quotient in quadratic form found
• Accept $x = 8$ without work if $f(1) = 0$ has been shown

Question 2 (c)

$$x = 1: r = \frac{x+8}{5x-8} = \frac{9}{-3} = -3$$

$$x = 8: r = \frac{16}{32} = \frac{1}{2}$$

S_{∞} exists for $|r| < 1$.

$$a = x^2 = 64, r = \frac{1}{2}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{64}{1-\frac{1}{2}} = 128$$

MARKING SCHEME NOTES**Question 2 (c) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- Substitution used to identify $x = 8$ as the required value
 - Substitution used to exclude $x = 1$ as the required value

- Finds $\frac{a}{1-r}$ for $x = 1$

- $S_{\infty} = \frac{x^2}{1 - \frac{5x-8}{x^2}}$

- Relevant substitution into correct formula
- 4:**
- GP identified (a and r)
 - If the candidate works with both $x = 1$ and $x = 8$ but fails to eliminate $x = 1$ or chooses the incorrect answer

Note: If $|r| > 1$ then Low Partial Credit at most