LC 2017 (Set A): Paper 1

Question 8 (55 marks)

Question 8 (a)

$$P=\frac{A}{(1+i)^1}+\frac{A}{(1+i)^2}+\frac{A}{(1+i)^3}+\ldots\ldots\ldots\ldots\ldots+\frac{A}{(1+i)^t}$$

$$a=\frac{A}{(1+i)},\ r=\frac{1}{(1+i)}$$

$$S_t=P=\frac{a(1-r^t)}{1-r}=\frac{\frac{A}{(1+i)}\left(1-\frac{1}{(1+i)^t}\right)}{1-\frac{1}{(1+i)}}=\frac{A\left(\frac{(1+i)^t-1}{(1+i)^t}\right)}{(1+i)\left(\frac{1+i-1}{(1+i)}\right)}$$

$$=\frac{A\left(\frac{(1+i)^t-1}{(1+i)^t}\right)}{i}=\frac{A((1+i)^t-1)}{i(1+i)^t}$$

$$\therefore A=P\frac{i(1+i)^t}{((1+i)^t-1)}$$

Question 8 (b) (i)

Monthly repayment $A=5000\times 0\cdot 025=€125$

Question 8 (b) (ii)

$\text{Annual rate}=21\cdot 75\% \Rightarrow i=0\cdot 2175$

$i_M=(1+i)^{\frac{1}{12}}-1=1\cdot 2175^{\frac{1}{12}}-1=0\cdot 0165$

$\text{Monthly rate}=1\cdot 65\%$

> $i=$ Annual interest rate
>
> $i_M=$ Monthly interest rate
>
> $i_W=$ Monthly interest rate
>
> $(1+i_M)^{12}=(1+i)\Rightarrow 1+i_M=(1+i)^{\frac{1}{12}}$
>
> $\therefore i_M=(1+i)^{\frac{1}{12}}-1$
>
> $\therefore i_W=(1+i)^{\frac{1}{52}}-1$

Question 8 (b) (iii)

You pay €125 every month. Part of this payment pays for the interest accrued that month and part of it pays off the overall debt.

End of month 1: Interest = €5000 × 0·0165 = €82·50
Balance reduced by: €125 – €82·50 = €42·50
New balance = €5000 – €42·50 = €4957·50

End of month 2: Interest = €4957·50 × 0·0165 = €81·80
Balance reduced by: €125 – €81·80 = €43·20
New balance = €4957·50 – €43·20 = €4914·30

End of month 3: Interest = €4914·30 × 0·0165 = €81·09
Balance reduced by: €125 – €81·09 = €43·91
New balance = €4914·30 – €43·91 = €4870·39

Payment Number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82·50	42·50	4957·50
2	125	81·80	43·20	4914·30
3	125	81·09	43·91	4870·39

Question 8 (b) (iv)

$P = 5000,\ A = 125,\ i_M = 0\cdot 0165,\ t = ?$

$A = P \times \dfrac{i_M(1+i_M)^t}{((1+i_M)^t - 1)} \Rightarrow 125 = 5000 \times \dfrac{0\cdot 0165(1\cdot 0165)^t}{(1\cdot 0165)^t - 1}$

$\dfrac{125}{5000 \times 0\cdot 0165} = \dfrac{(1\cdot 0165)^t}{(1\cdot 0165)^t - 1}$

$\dfrac{50}{33} = \dfrac{1\cdot 0165^t}{1\cdot 0165^t - 1} \Rightarrow 50(1\cdot 0165^t - 1) = 33 \times 1\cdot 0165^t$

$50 \times 1\cdot 0165^t - 50 = 33 \times 1\cdot 0165^t$

$17 \times 1\cdot 0165^t = 50$

$1\cdot 0165^t = \frac{50}{17} \Rightarrow \log_{10} 1\cdot 0165^t = \log_{10} \frac{50}{17}$

$t \log_{10} 1\cdot 0165 = \log_{10} \frac{50}{17}$

$\therefore t = \dfrac{\log_{10} \frac{50}{17}}{\log_{10} 1\cdot 0165} = 65\cdot 92$

Answer: 66 months

Question 8 (b) (v)

$i = 0\cdot 085$

$i_W = (1+i)^{\frac{1}{52}} - 1 = (1\cdot 085)^{\frac{1}{52}} - 1 = 0\cdot 00157$

$P = 5000,\ i_W = 0\cdot 00157,\ t = 156 \text{ weeks},\ A = ?$

$A = P \times \dfrac{i_W(1+i_W)^t}{((1+i_W)^t - 1)} \Rightarrow A = 5000 \times \dfrac{0\cdot 00157(1\cdot 00157)^{156}}{(1\cdot 00157)^{156} - 1} = €36\cdot 16$

Question 8 (b) (vi)

$\text{Saving} = 125 \times 66 - 36\cdot 16 \times 156 = €2609\cdot 04$