LC 2017 (Set A): Paper 1

Question 5 (25 marks)

Question 5 (a)

$f(x) = 2x^3 + 5x^2 - 4x - 3$

$f(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3 = -54 + 45 + 12 - 3 = 0$

$\therefore (x+3)$ is a factor

$2x^3 + 5x^2 - 4x - 3 = (x+3)(2x^2 + kx - 1)$

$= 2x^3 + (k+6)x^2 + \ldots$

$5 = k + 6 \Rightarrow k = -1$

$\therefore 2x^3 + 5x^2 - 4x - 3 = (x+3)(2x^2 - x - 1) = 0$

$2x^2 - x - 1 = (2x+1)(x-1) = 0 \Rightarrow x = -\frac{1}{2}, 1$

$$
\begin{array}{r|l}
 & 2x^2 - x - 1 \\
\hline
x + 3 & 2x^3 + 5x^2 - 4x - 3 \\
 & \mp 2x^3 \mp 6x^2 \\
\hline
 & -x^2 - 4x - 3 \\
 & \pm x^2 \pm 3x \\
\hline
 & -x - 3 \\
 & \pm x \pm 3 \\
\hline
 & 0
\end{array}
$$

Question 5 (b)

$f(x) = 2x^3 + 5x^2 - 4x - 3$

$f'(x) = 6x^2 + 10x - 4$

$f''(x) = 12x + 10$

Turning points: $f'(x) = 0 \Rightarrow 6x^2 + 10x - 4 = 0$

$3x^2 + 5x - 2 = 0$

$(3x-1)(x+2) = 0 \Rightarrow x = \frac{1}{3}, -2$

$f(\frac{1}{3}) = 2(\frac{1}{3})^3 + 5(\frac{1}{3})^2 - 4(\frac{1}{3}) - 3 = -\frac{100}{27}$

$f(-2) = 2(-2)^3 + 5(-2)^2 - 4(-2) - 3 = 9$

$f''(\frac{1}{3}) = 12(\frac{1}{3}) + 10 = 14 > 0 \Rightarrow (\frac{1}{3}, -\frac{100}{27})$ is a local minimum

$f''(-2) = 12(-2) + 10 = -14 < 0 \Rightarrow (-2, 9)$ is a local maximum

Question 5 (c)

$f(x) + a$: Roots are on opposite sides of x-axis

Translate $f(x)$ **up** by $\frac{100}{27} \Rightarrow a > \frac{100}{27}$

Translate $f(x)$ **down** by $9 \Rightarrow a < -9$

$\therefore a < -9,\ a > \frac{100}{27},\ a \in \mathbb{R}$